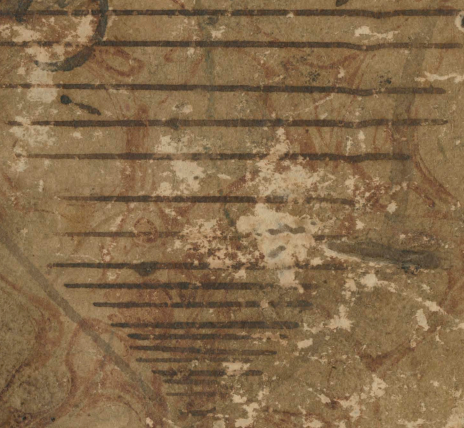


Alfred [illegible]
Fidelity & [illegible]
Fidelity



MORISS-50162

[99.009]

E. 124

Aug-1855-

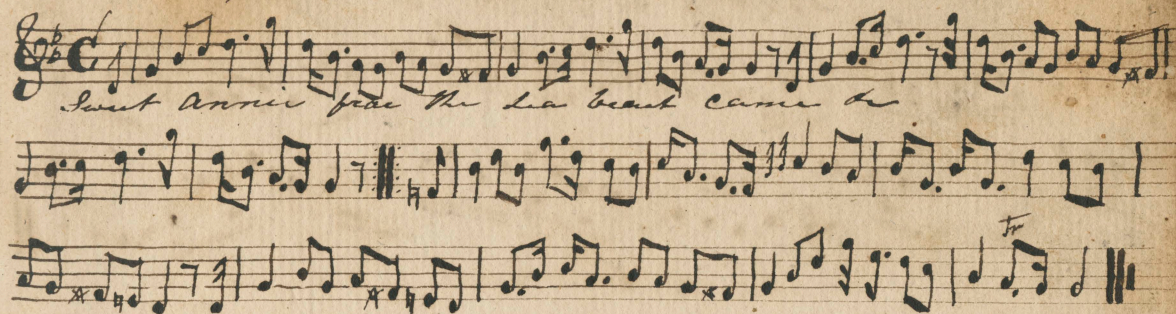
Buchanan's Journal of Man is devoted to the Science of Anthropology
and is published at Cincinnati

Book of Milton Barlow
bought 1784. (See his faint signature
at top of front board)

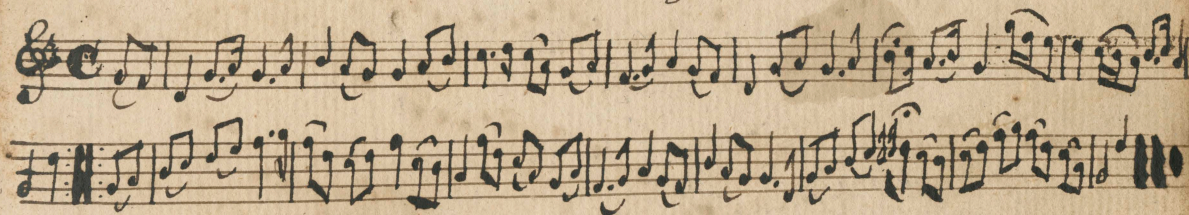
11/24/85

bought of a Chestnut Hill Mass dealer
1885. RHF

Sweet Annie



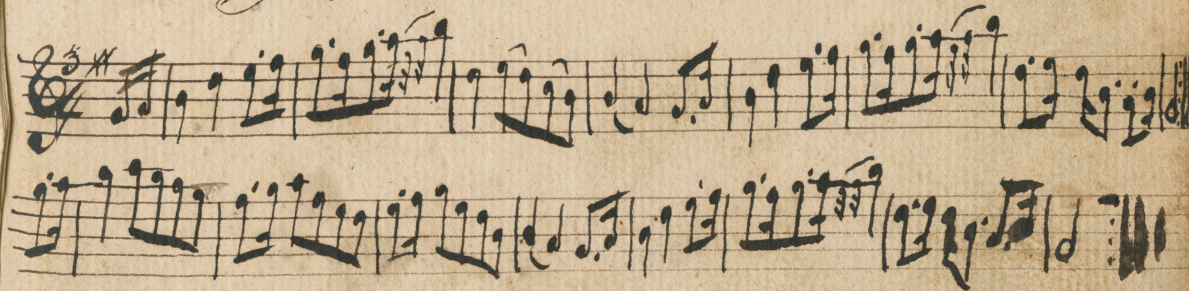
Highland Mary -



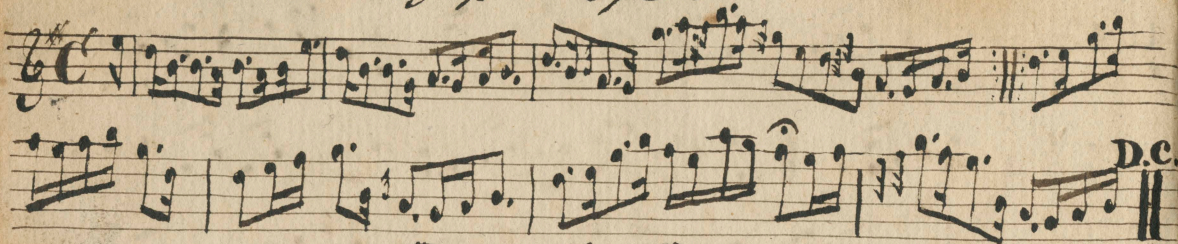
Coming thro' the Rye



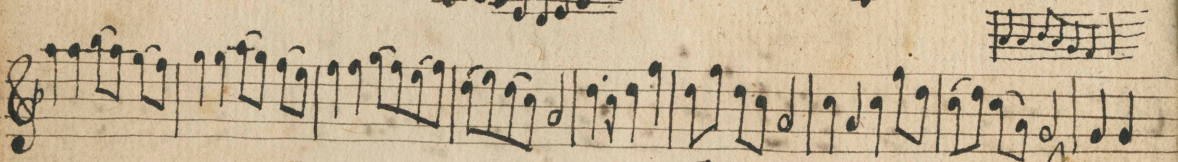
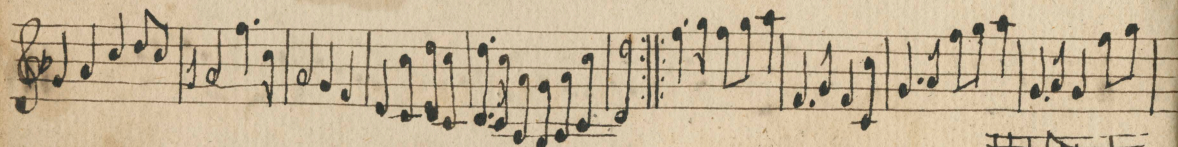
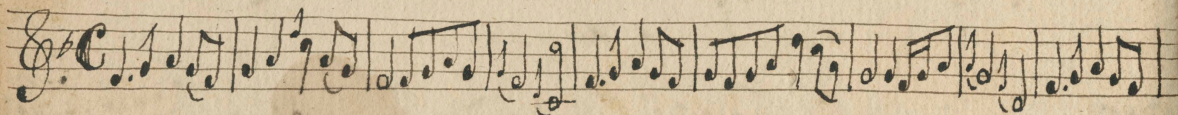
Yellow Haired Laddie



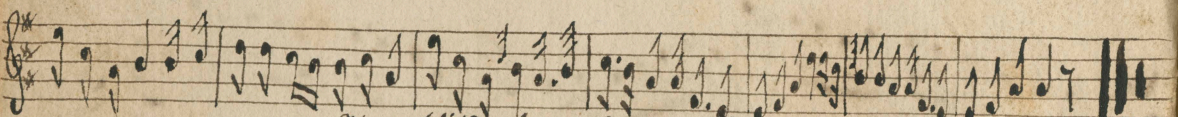
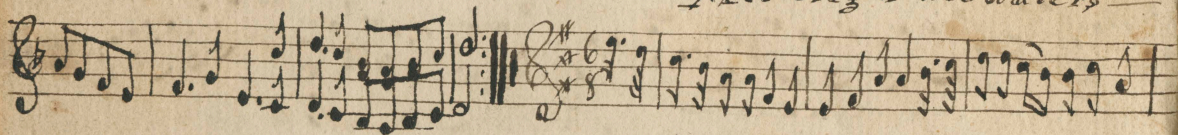
~~Contra Alt. B.~~
 Boy's wife. D



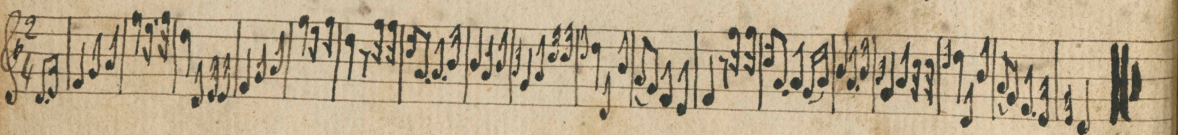
Port Coraton



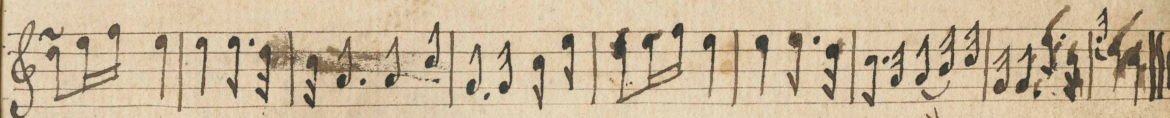
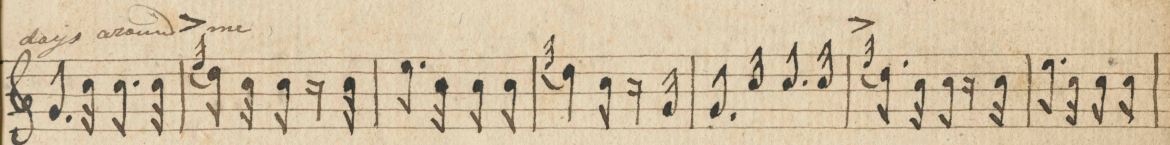
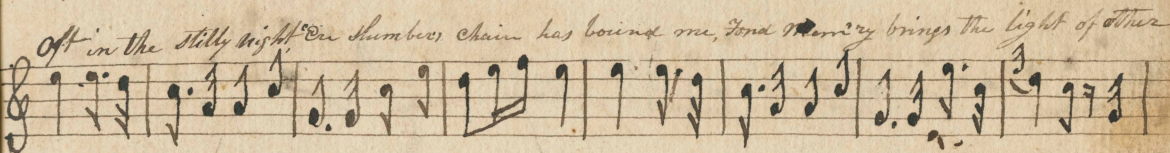
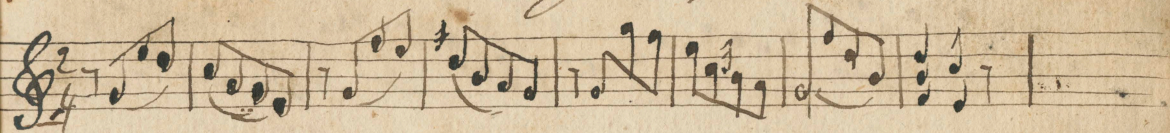
Meeting of the waters



How blithe have I been with my Landy



oft in the Stillly Night



oft in the Stillly Night.

Our Slumbers Chain has bound me
Fond Memory brings the light
Of other days around me

The smiles the tears
Of Boyhood's years
The words of love then spoken

The eyes that shone
Now dim'd & gone

The cheerful hearts now broken

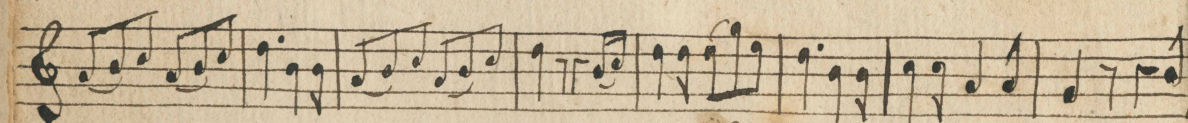
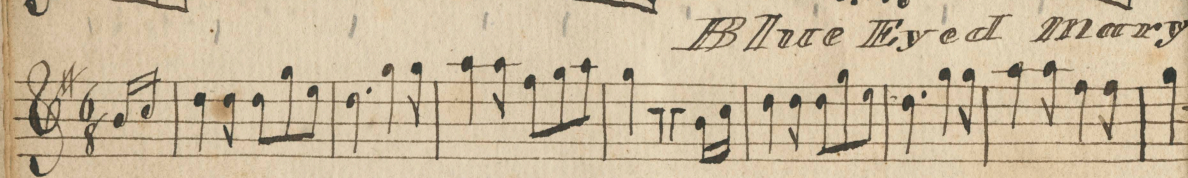
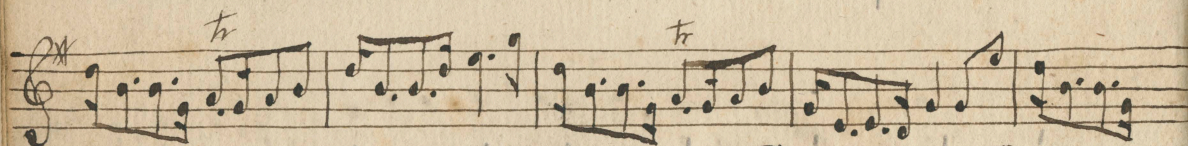
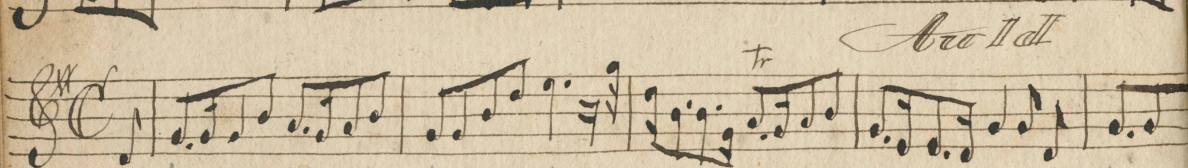
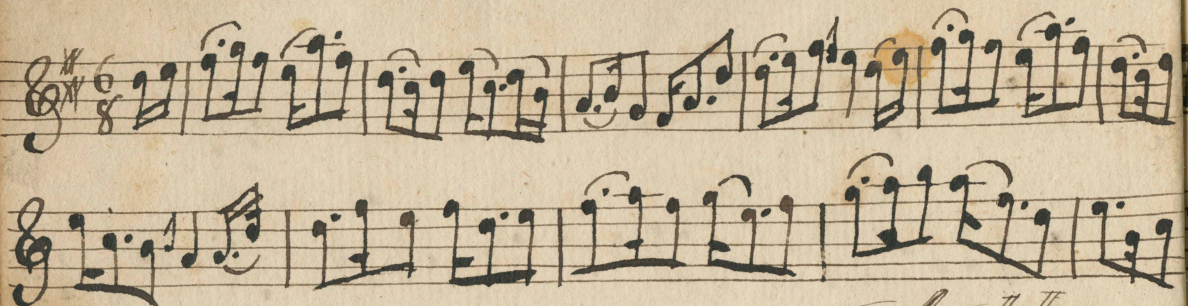
They in the Stillly night, Our Slumbers chain has bound me
Sad memory brings the light, Of other days around me

When I remember all, the friends so linked together
So soon around my path, like leaves in winter weather,
I feel like one, who lonely alone
Some banquet table deserted
Whose lights are put, whose gaily dance
And see but be departed -

They in the Stillly Night
Our Slumbers Chain has bound me
Sad memory brings the light
Of other days, around me

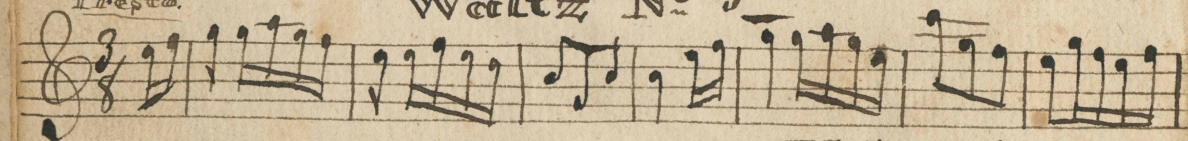
4

Sessie the



Presto.

Waltz No 3



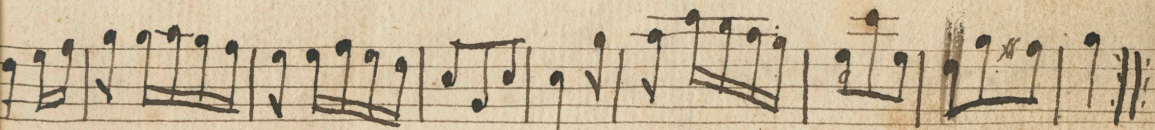
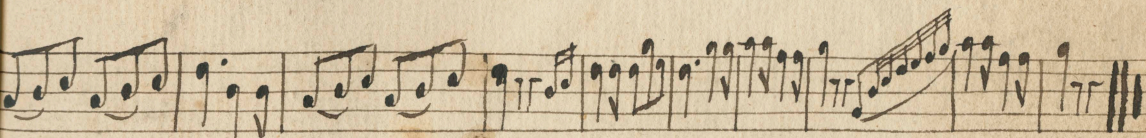
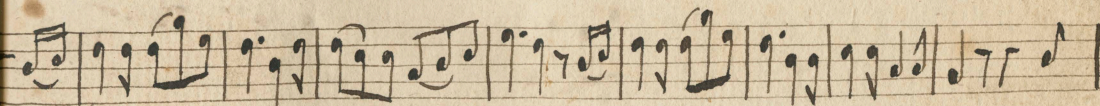
Waltz -

Flower of Dumb Lane

५



Ther 22 g Sty 22 c.

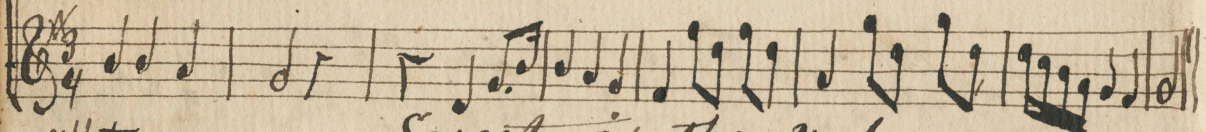
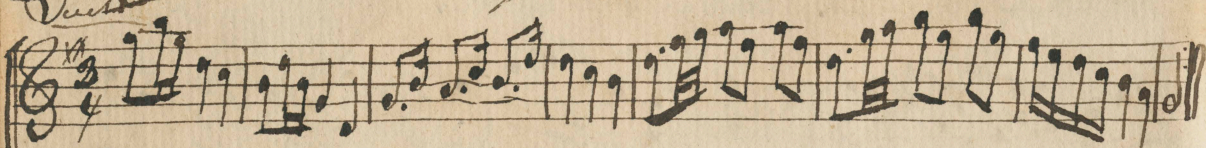


No. 3



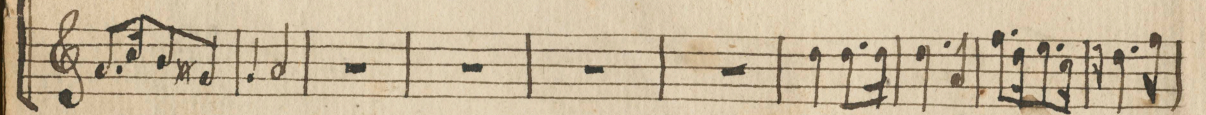
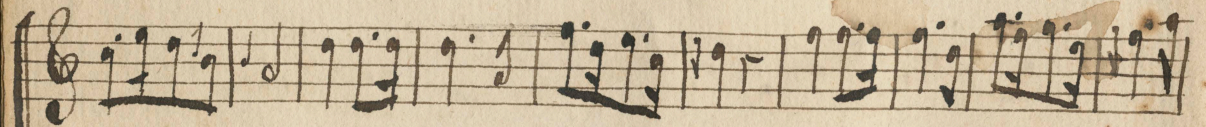
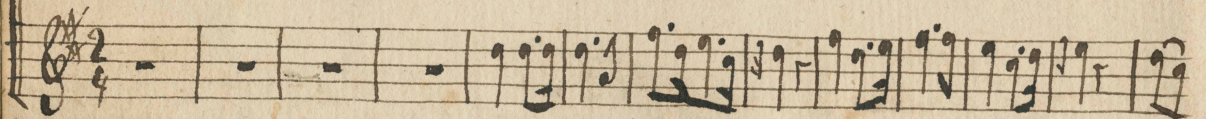
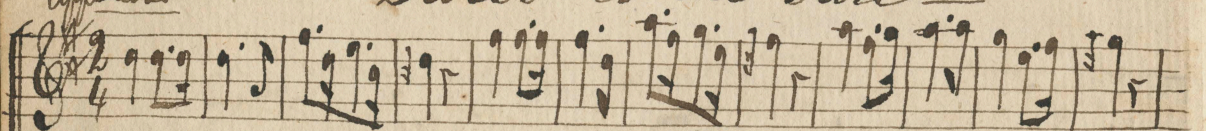
Quint

Ambassador's Minuet.

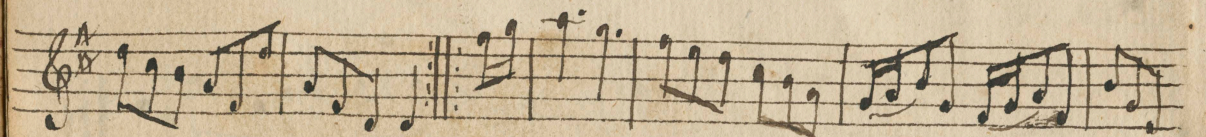
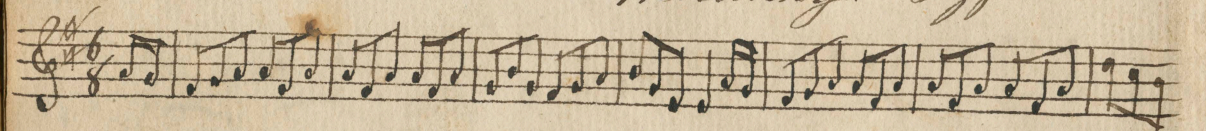


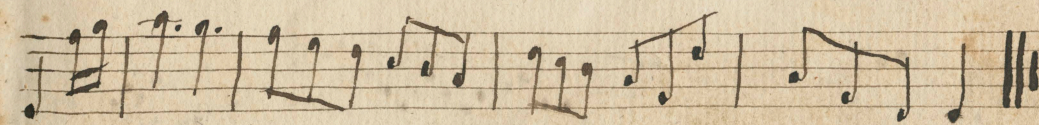
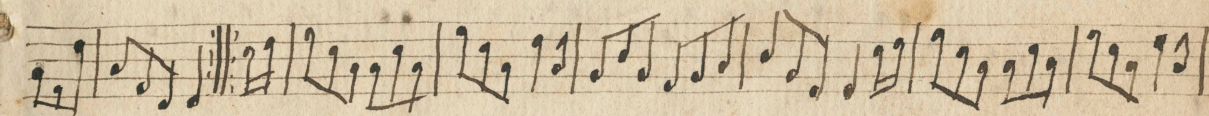
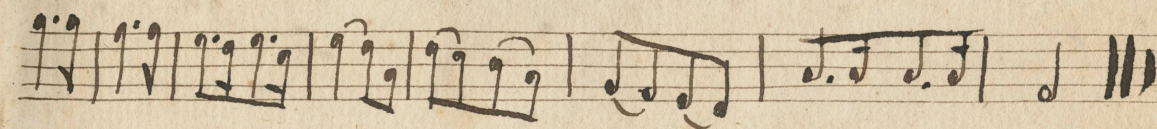
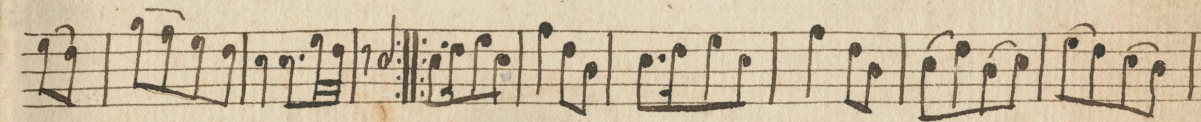
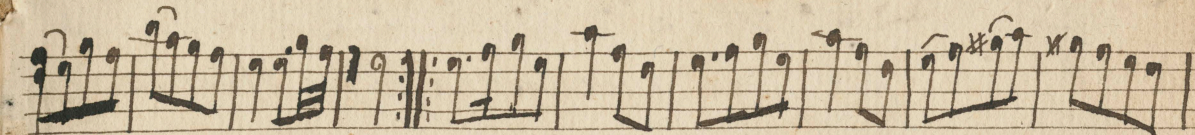
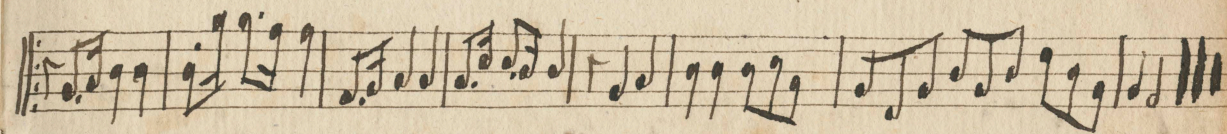
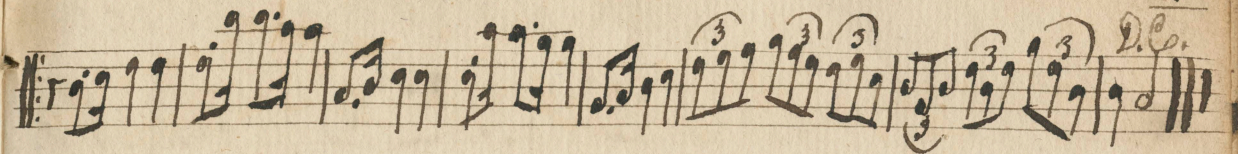
affettuoso

Sweet is the vale —

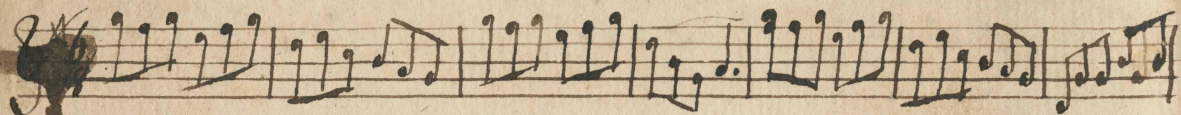


Mollonny's Jigg —

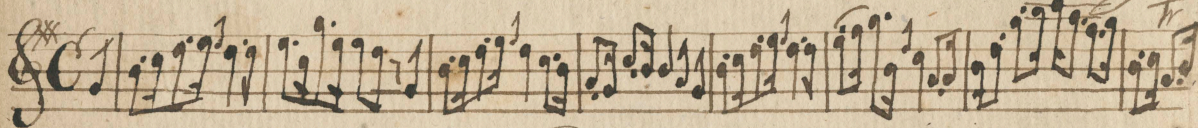




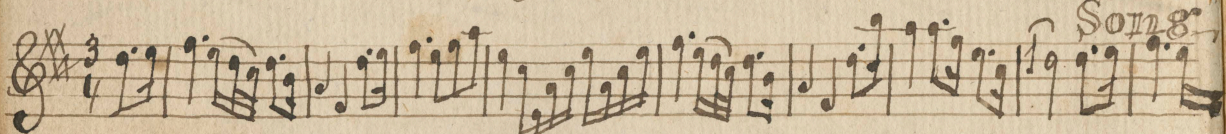
Jackson's Welcome



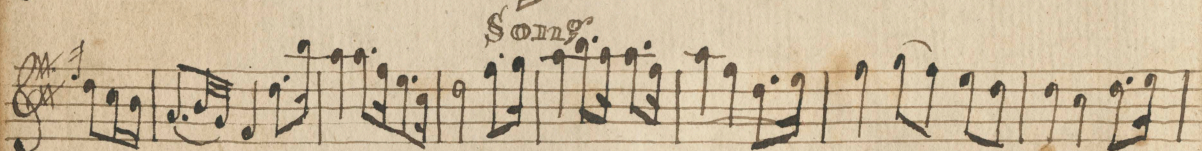
Auld Robin Gray



Durandarte & Belerma

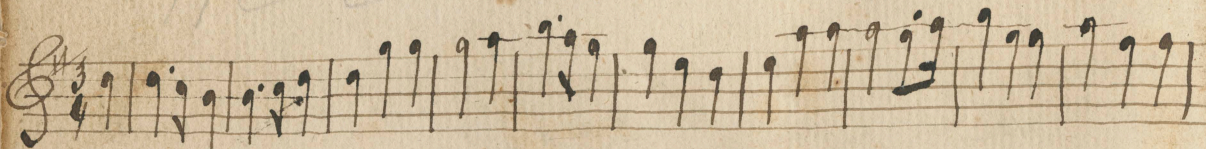


Song

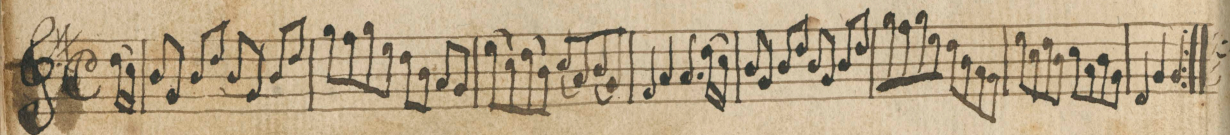


Song

Sandy & Tenny

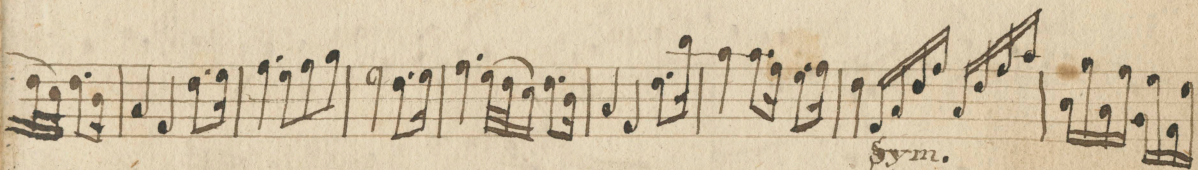


Alti Ps Alti am

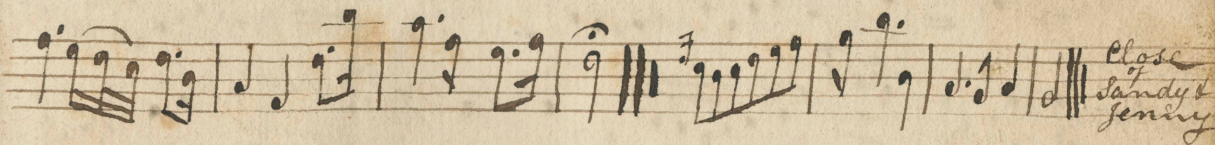


e Home

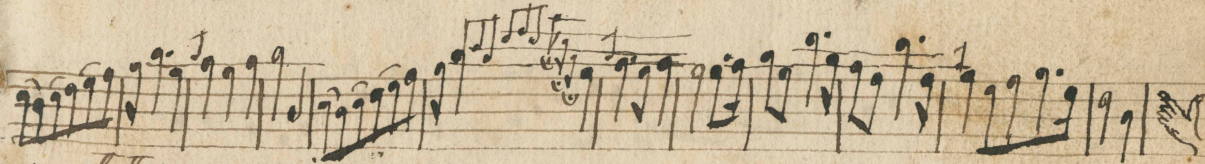
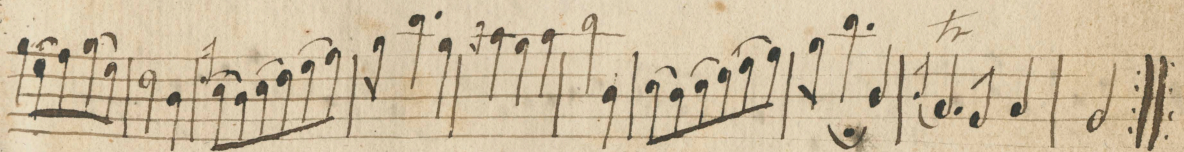
9



Sym.

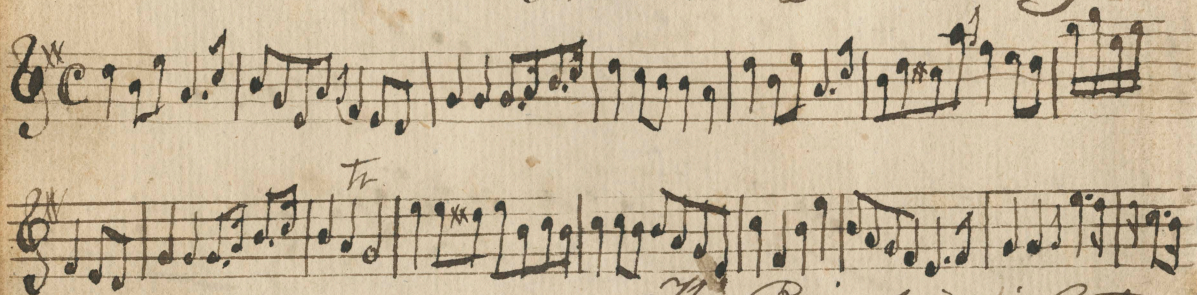
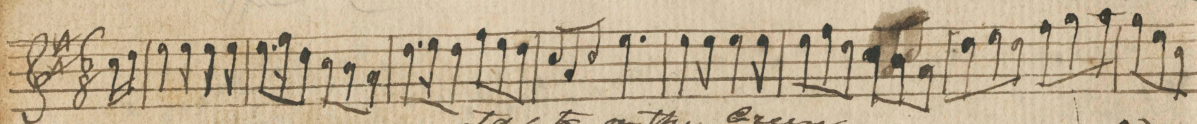
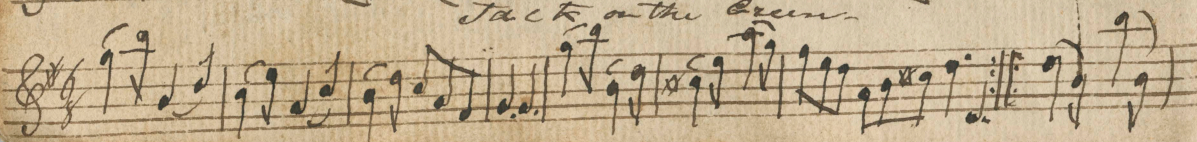


Close
Sandy
Jenny



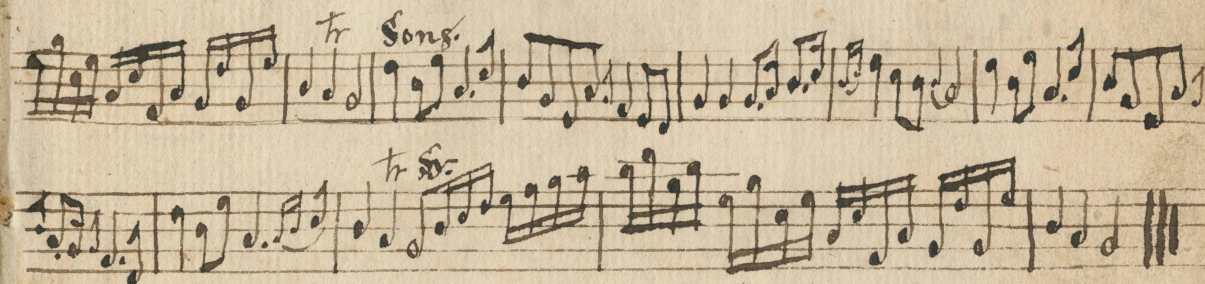
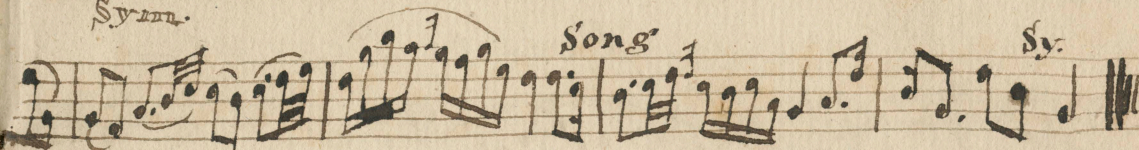
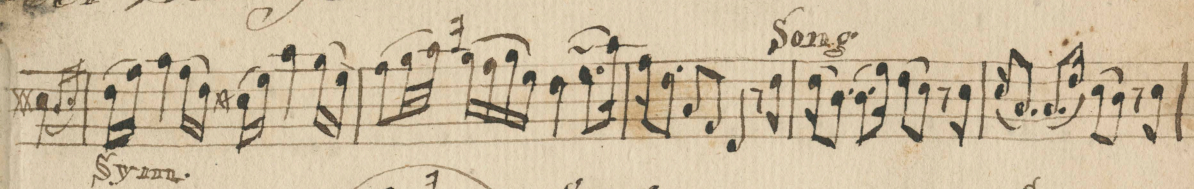
1 Horn pipe

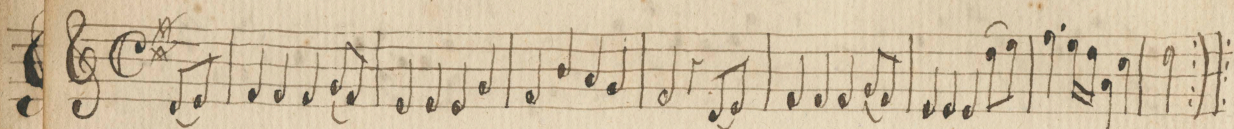
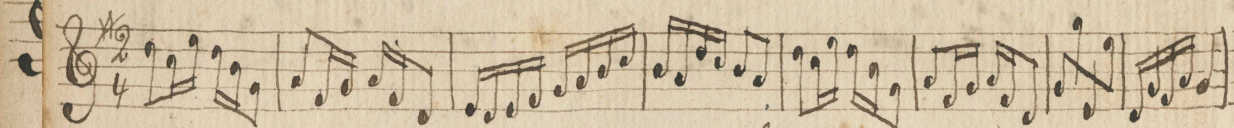
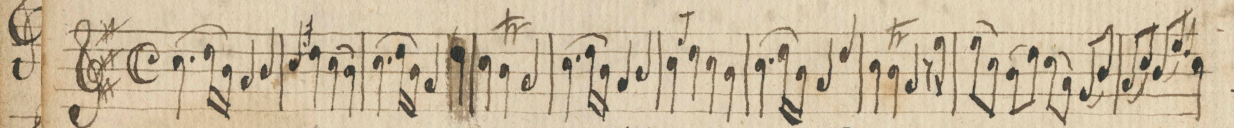
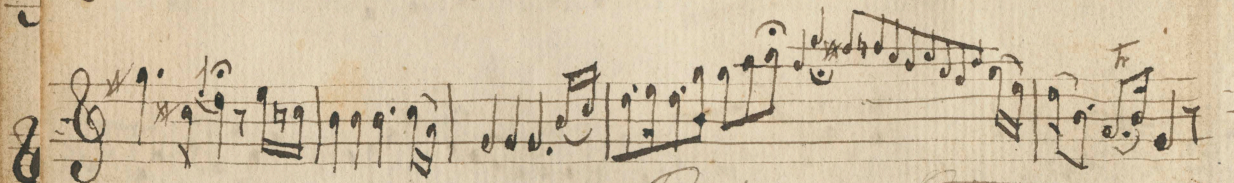
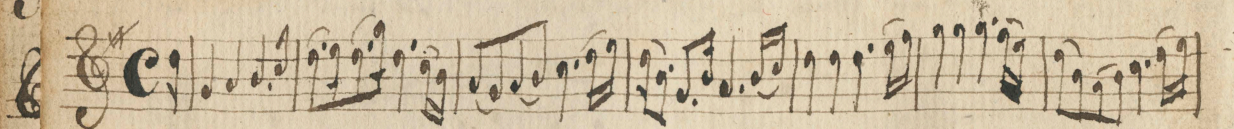
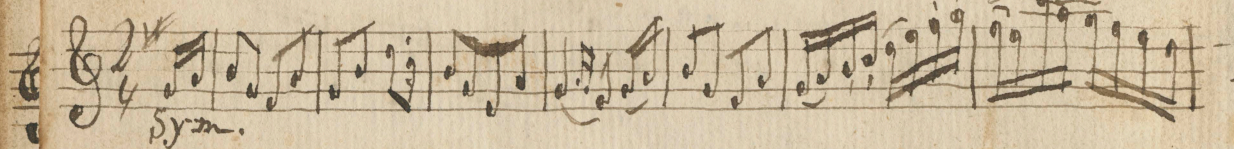


Shepherd's Dance*The Cottage on the Moon**When Wars Alarm**The Priest in his Bells**Jack on the Green*

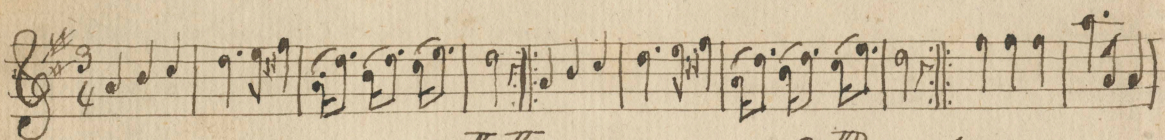
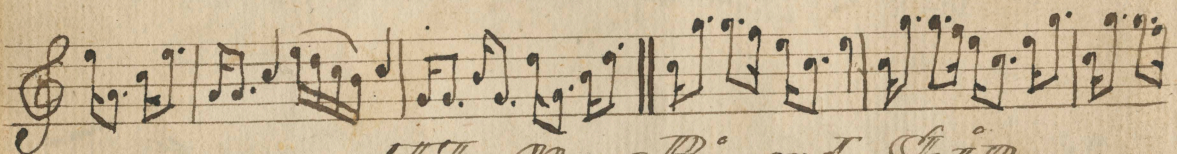
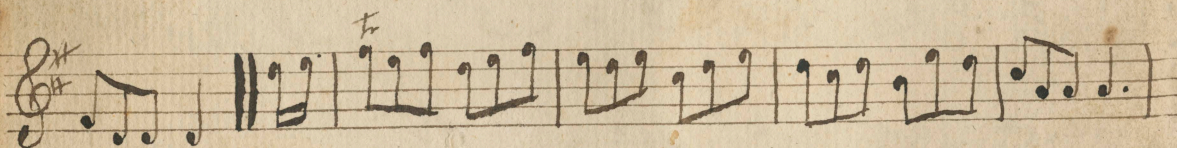
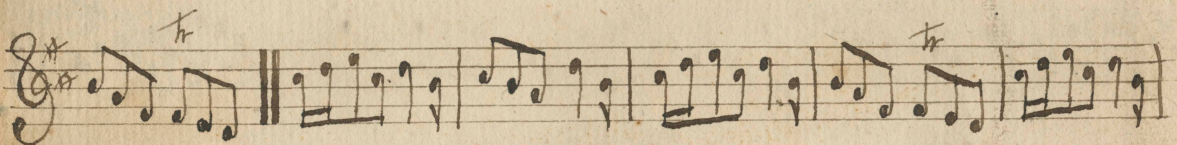
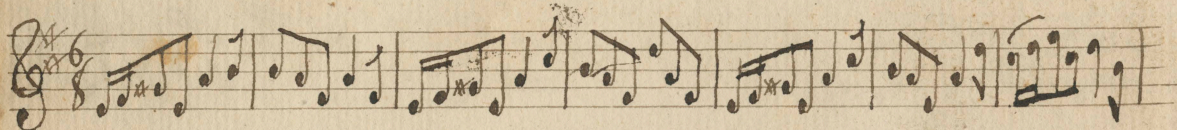
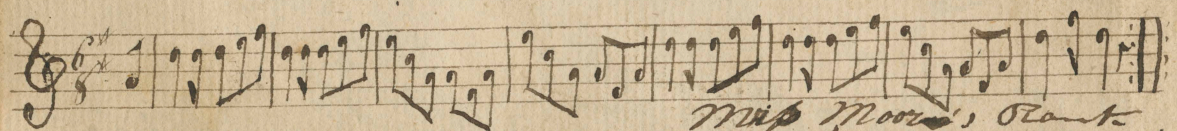
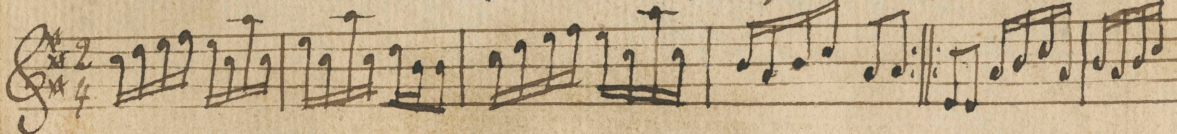
ter Sally-

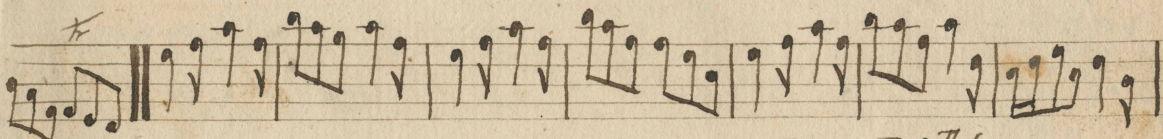
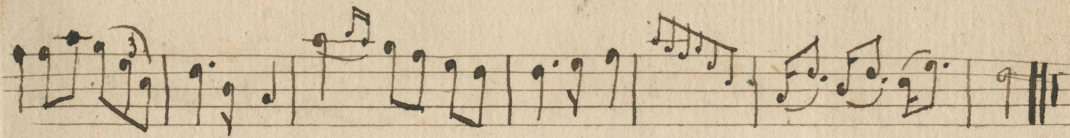
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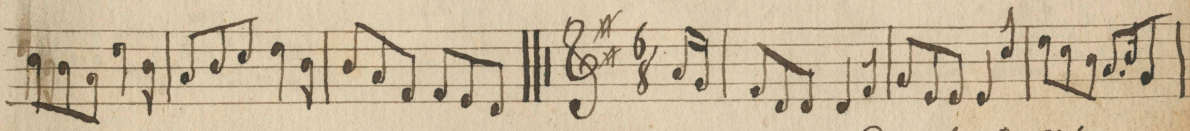
Blodget's Hymn*Prince Regent**No 'tis neither Shape, nor Feature**Mary's Dream**Contented Cottager*



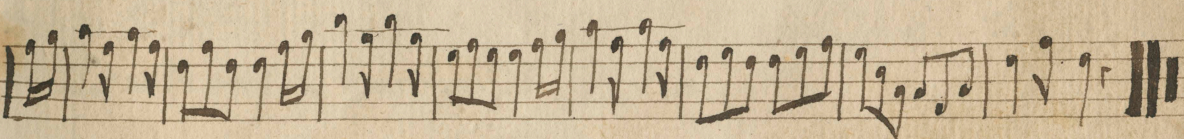
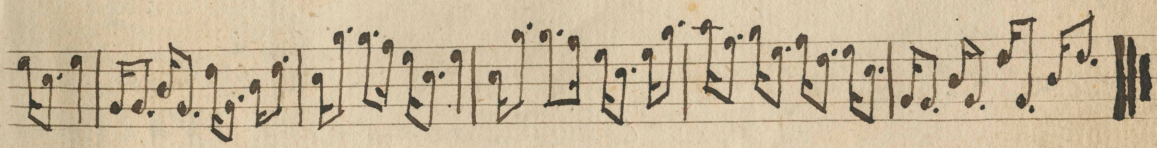
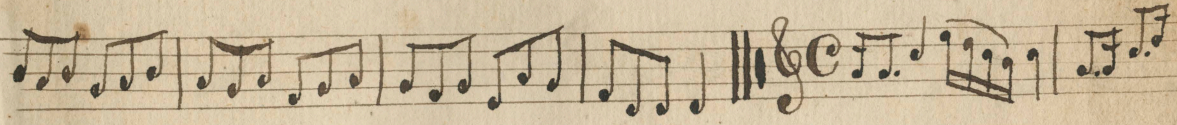
Robin Adier.Humours of Parateen.The New Rigg'd Ship.Miss Moore's Plant.

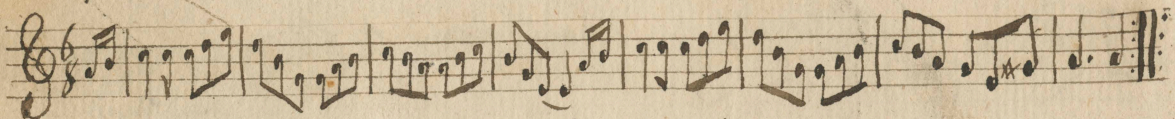
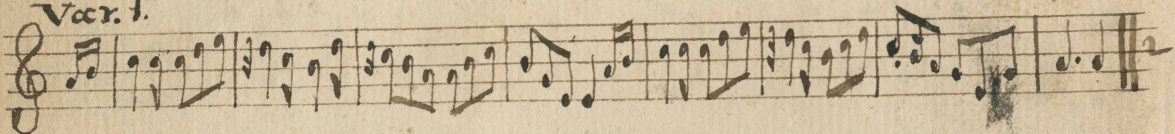
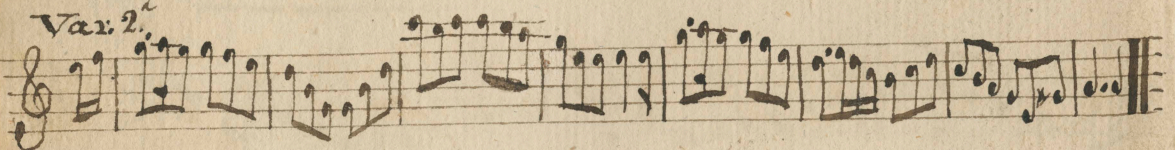


Irish Lilt.

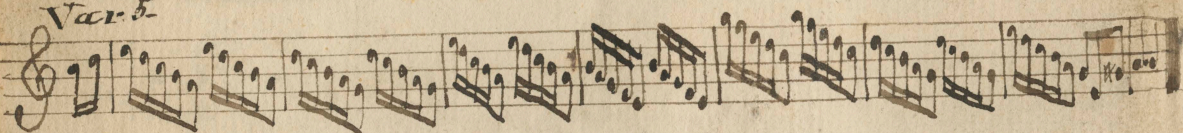


A Scotch air

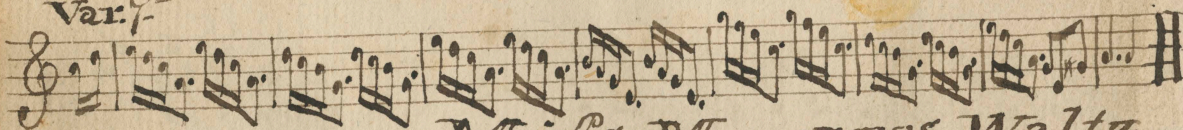
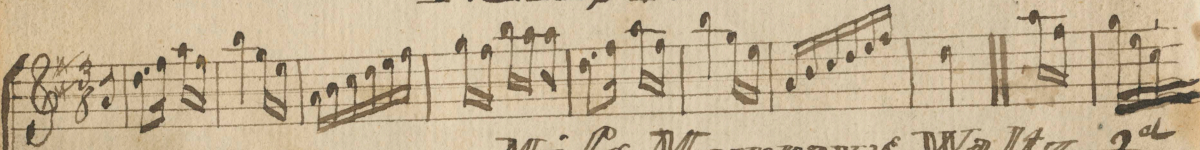
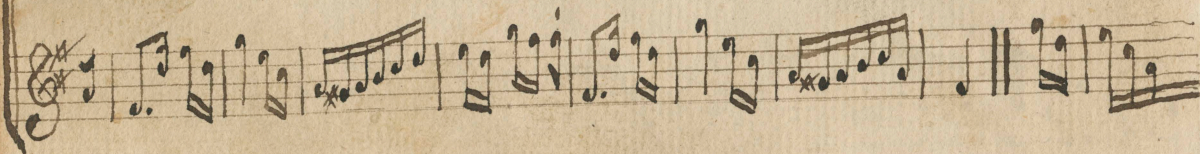


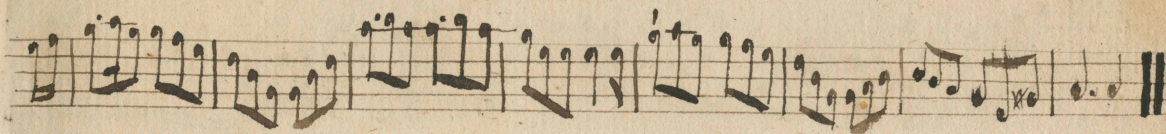
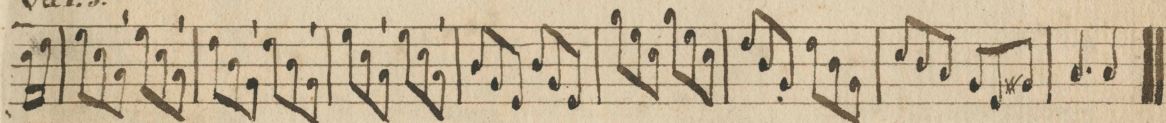
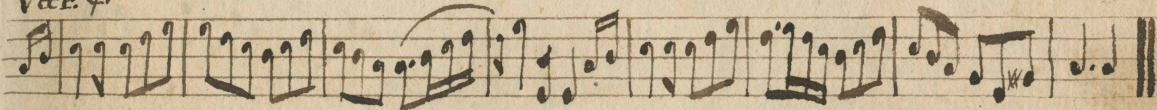
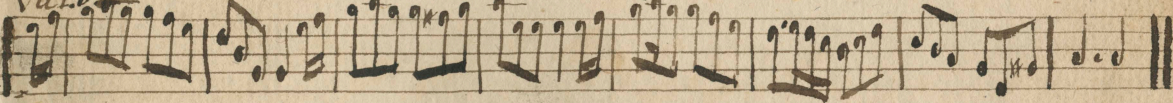
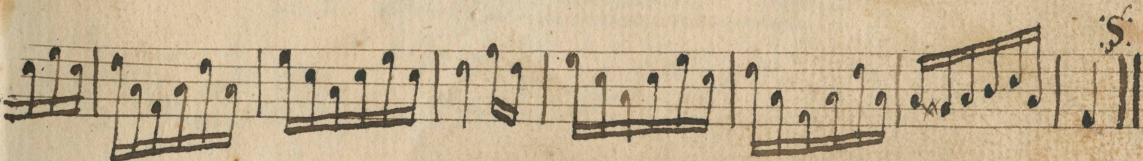
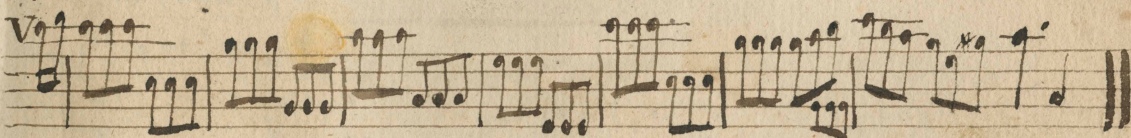
Green Sleeves, with variations.Var. 1stVar. 2nd

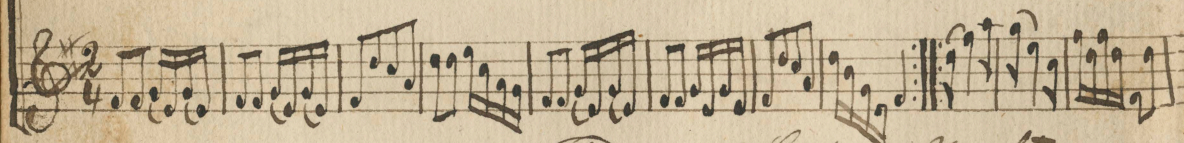
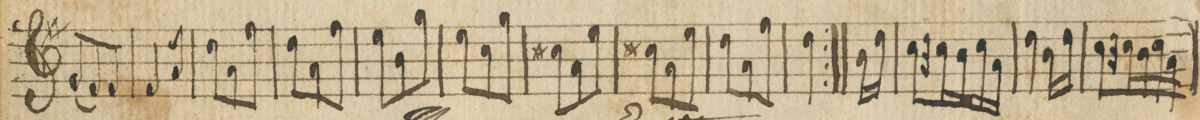
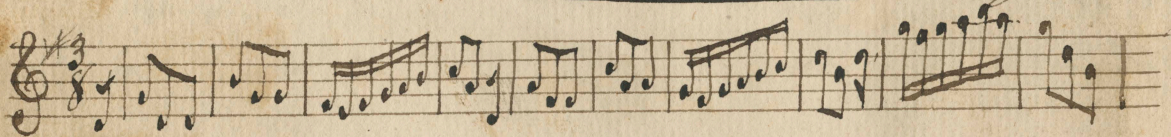
Var. 5



Var. 7

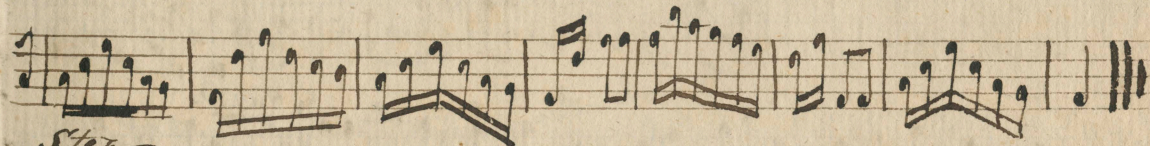
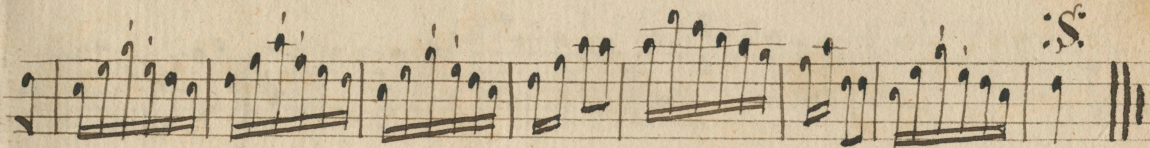
*Miss Murray's Waltz.**Miss Murray's Waltz. 2nd*

Var. 3^dVar. 4thVar. 6thVar. 8₂

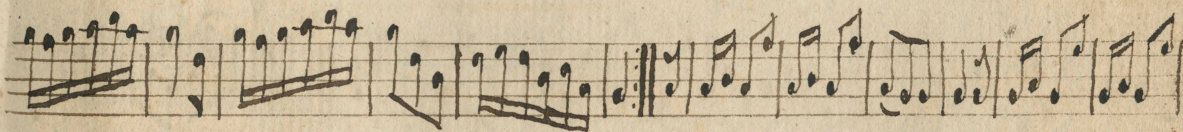
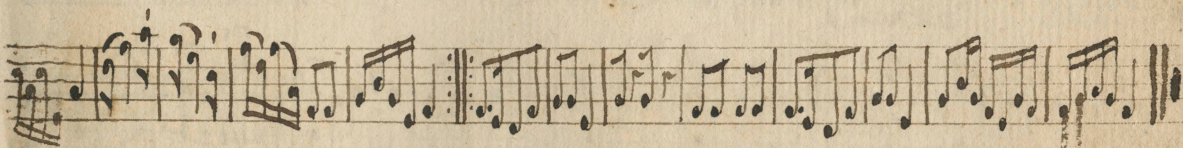
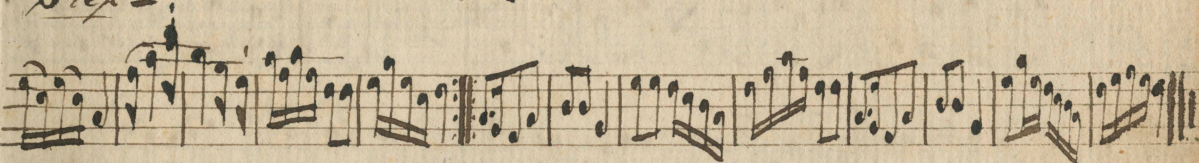
Miss Murray's Wally.Duke of York's QuickPrince Leopold's Wally.The Opera Editor

Continued -

19



Step -



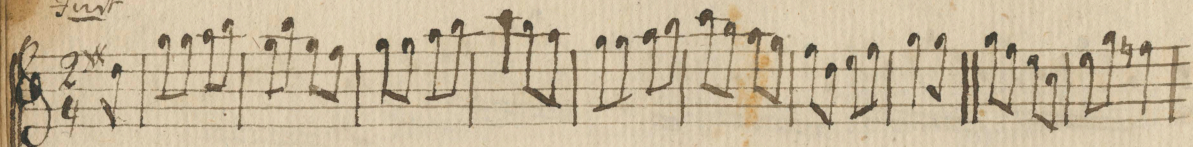
Augustine's Waltz



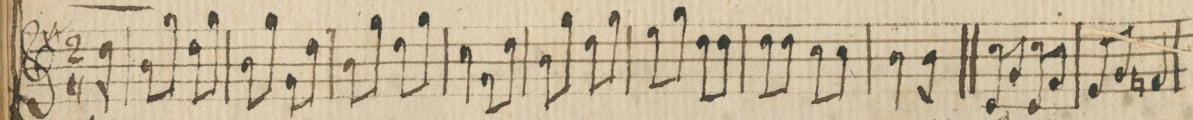
20

First

Yan Kee Doodle. with Variations

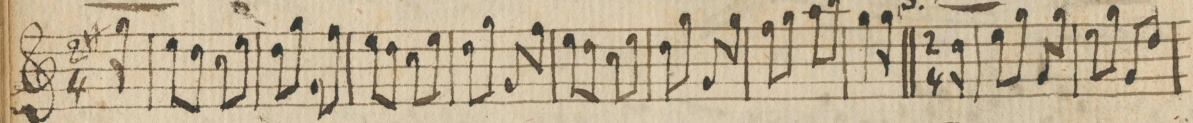


Second



Var. 2.

S. Var. 3.

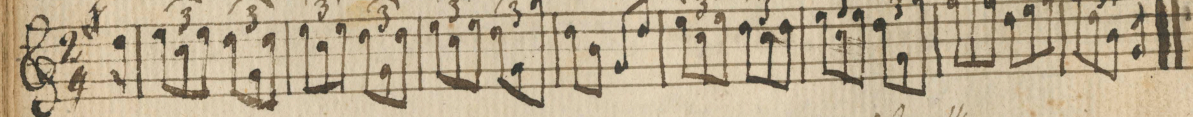


S. Var. 4.

D.C. Var. 1.

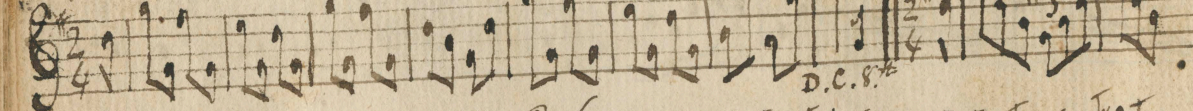


Var. 9.



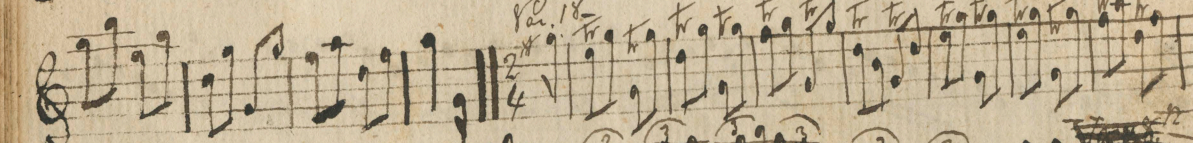
Var. 13.

Var. 14.



D.C. 8.

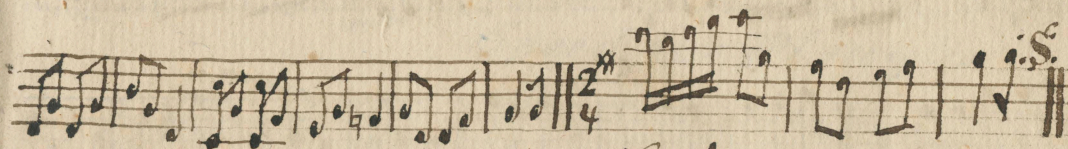
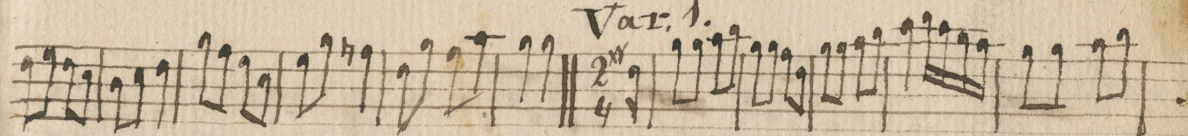
Var. 18.



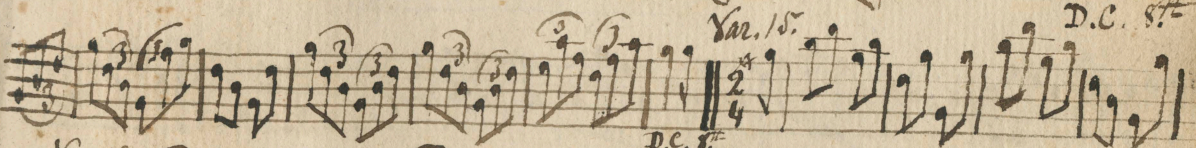
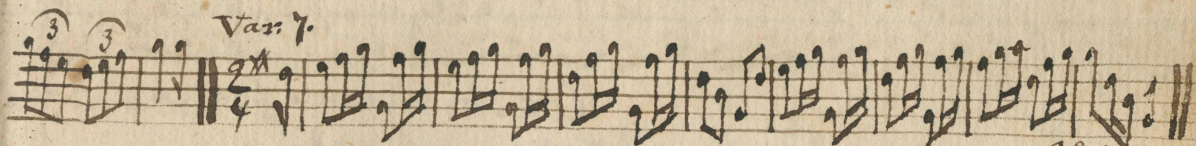
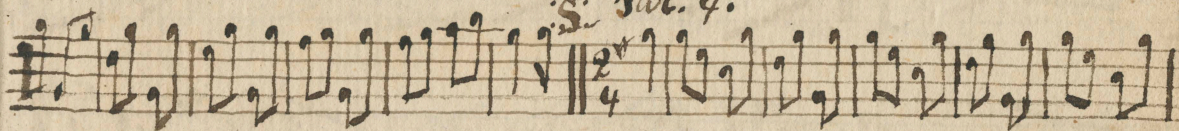
Var. 20



Var. 1.



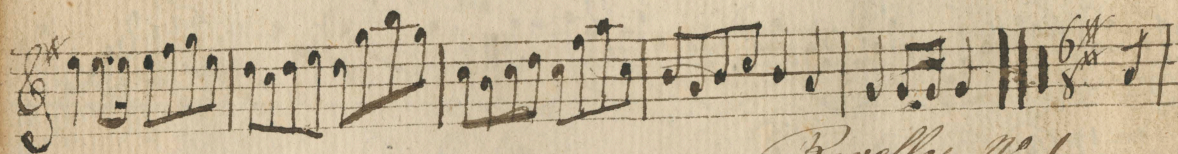
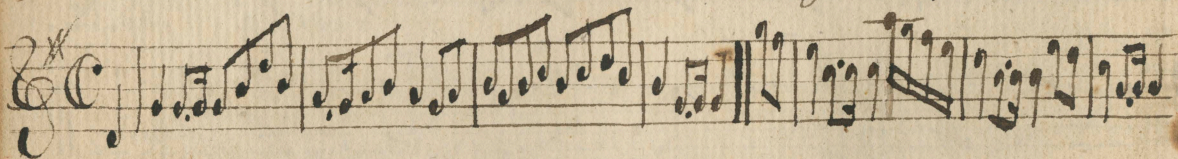
S. Var. 4.



Devils Dream.



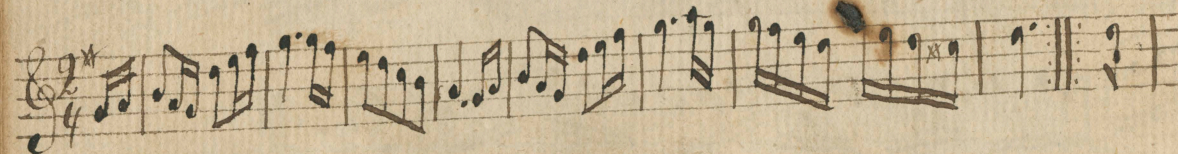
Washington's Grand

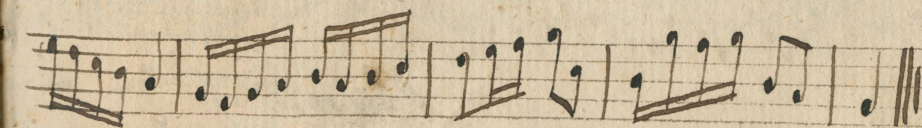
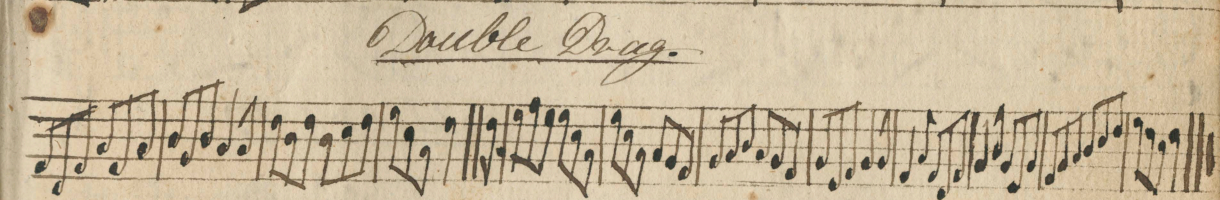
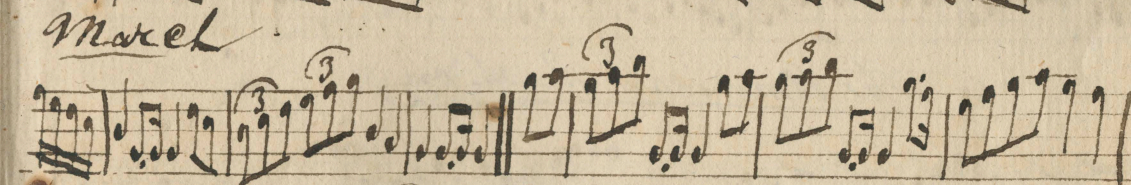
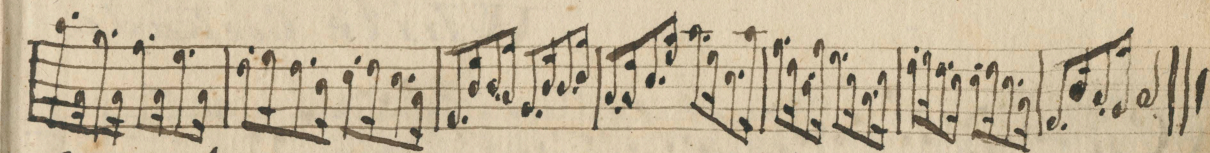
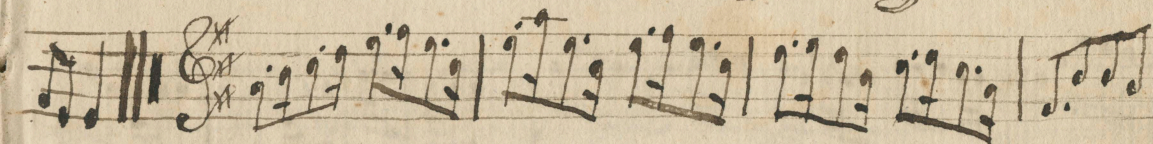


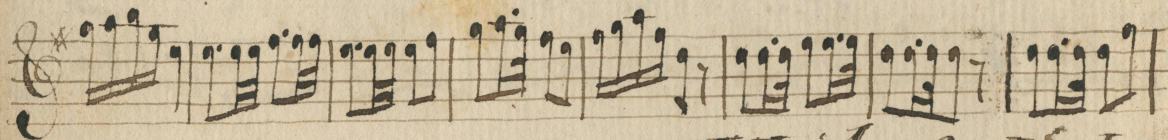
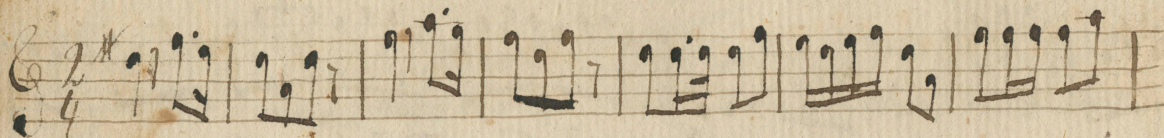
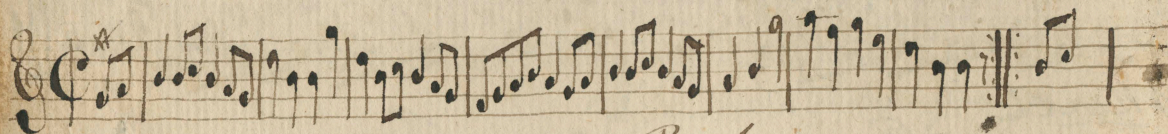
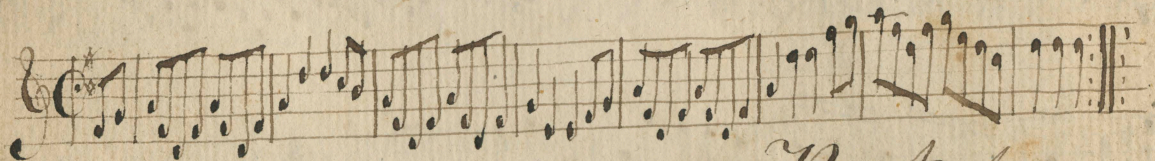
Revelly. No. 1.



Retreat No. 1.

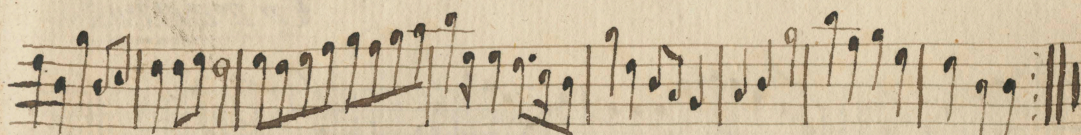
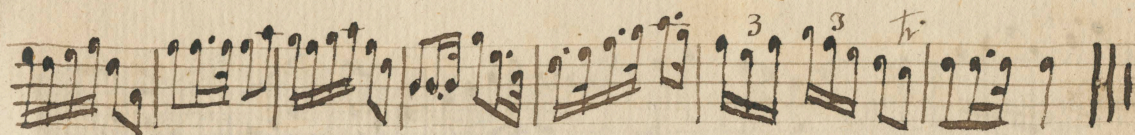
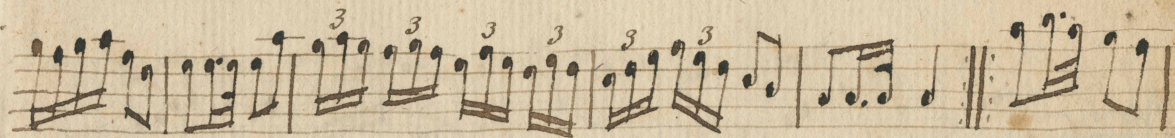




Hail !!!White CockadeThe American Rush onSoldiers JoyBank of - -

Coltrane Hwa.

25

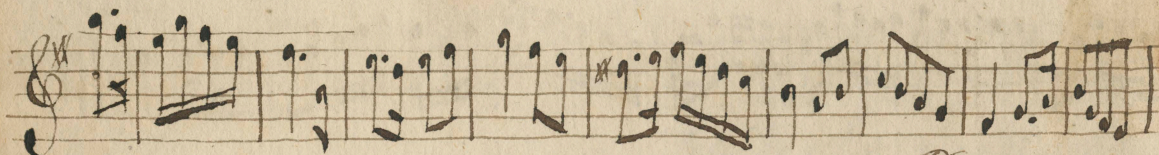
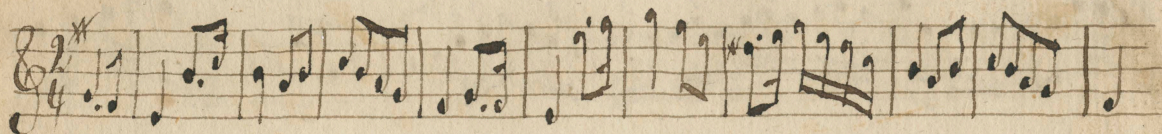
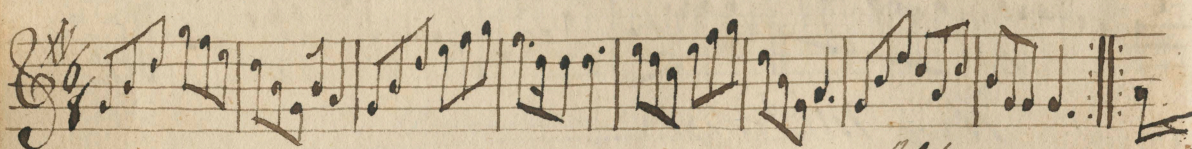
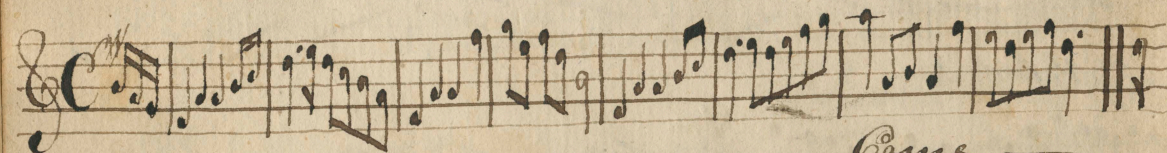


Hay McKing.



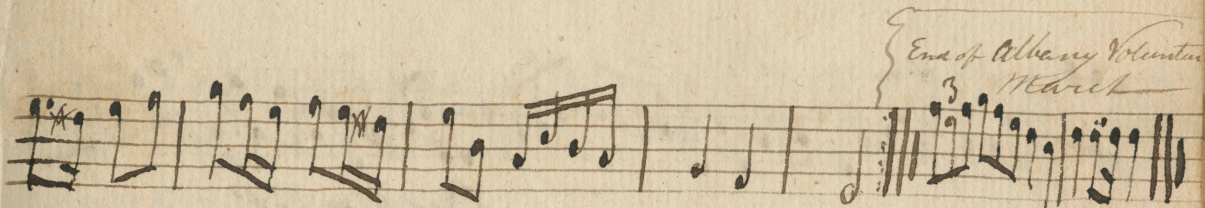
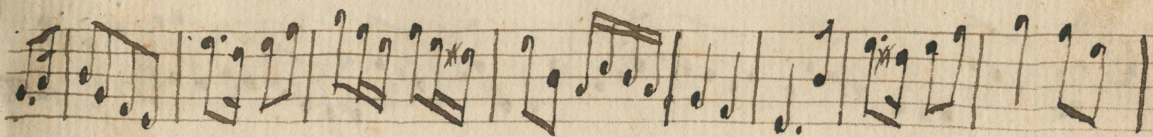
Flowers



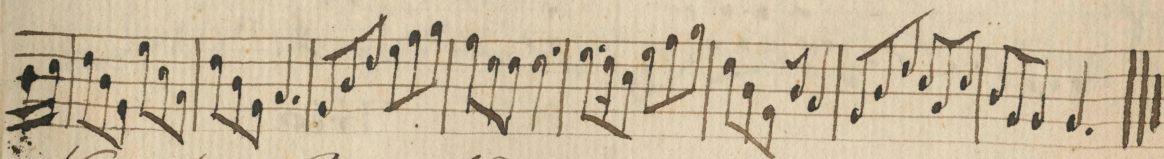
GreenManyConstitu-Come

Castle.

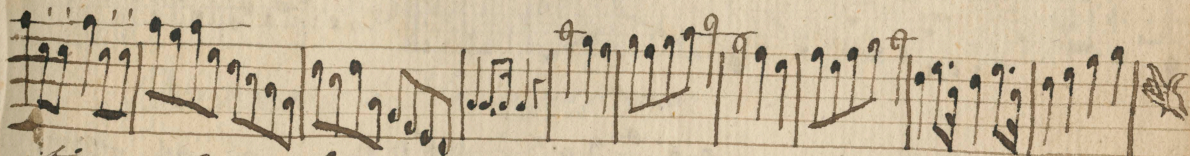
27



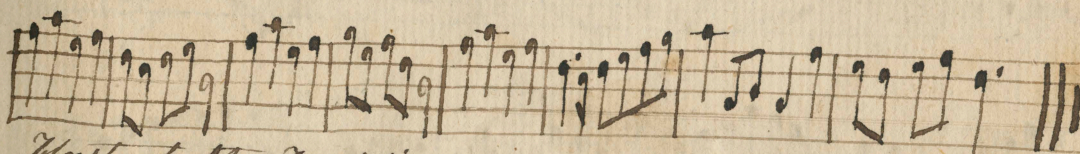
Joke.



Volunteers March.

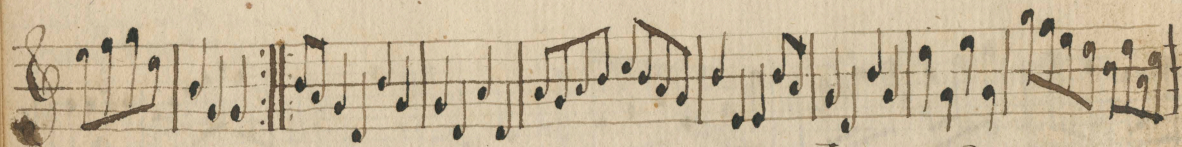
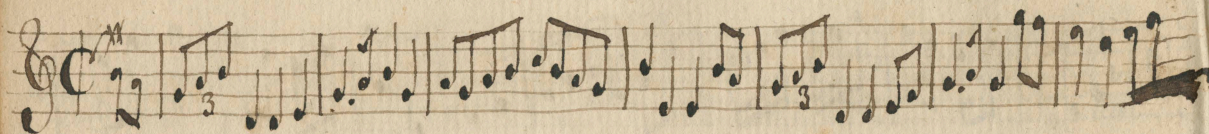


-tion March.

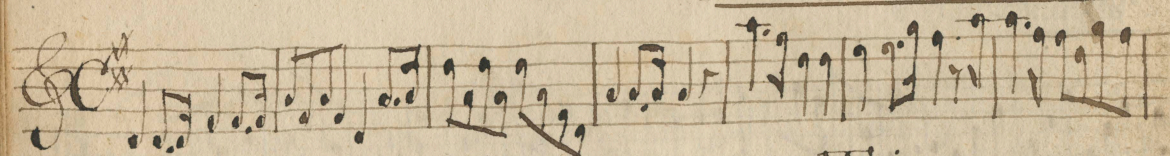


Haste to the Wedding.

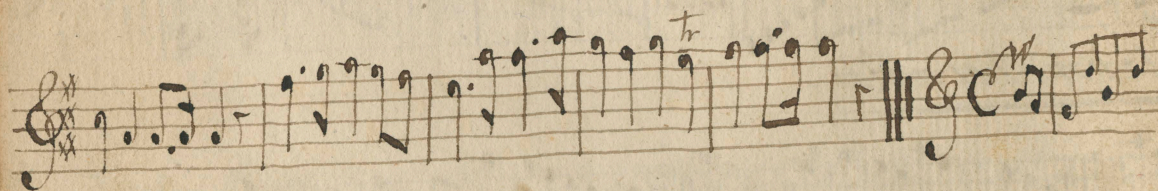




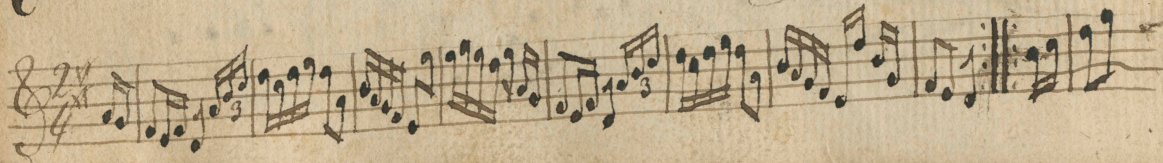
Duke of



Trio.



The American



March

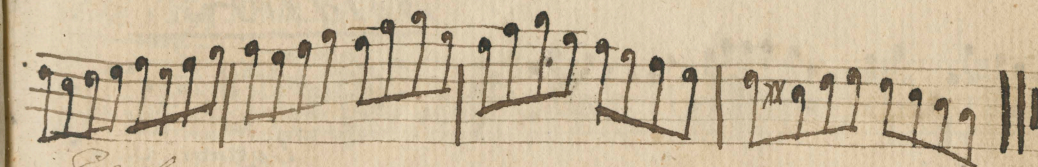
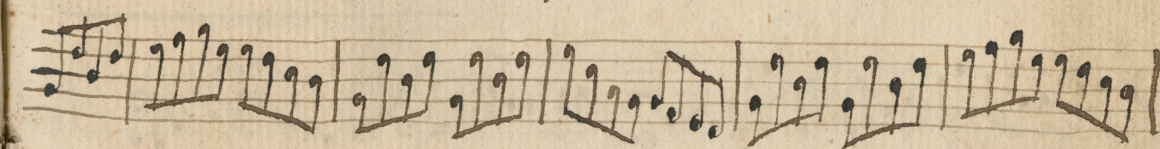
29



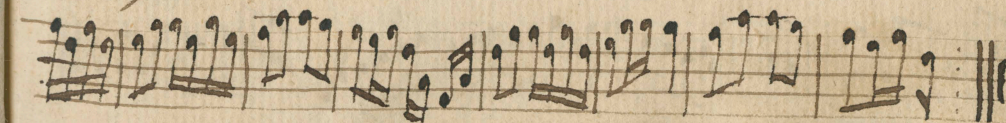
York's march

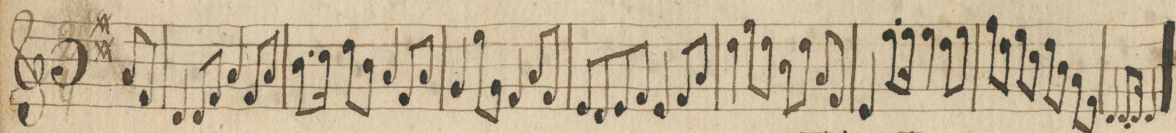
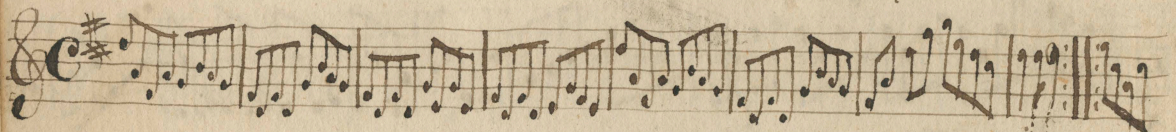
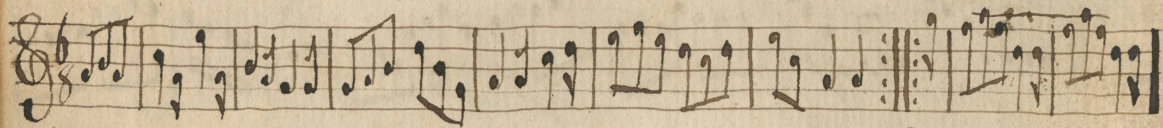
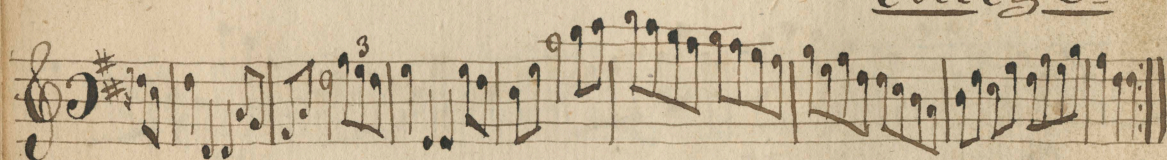
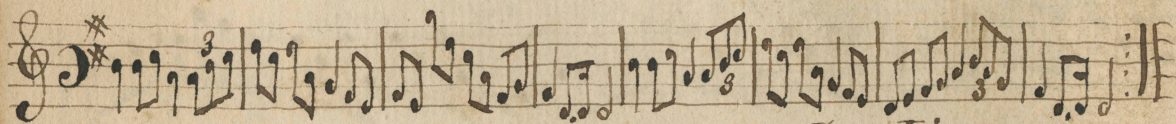
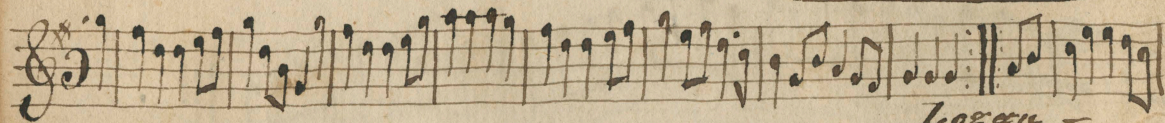
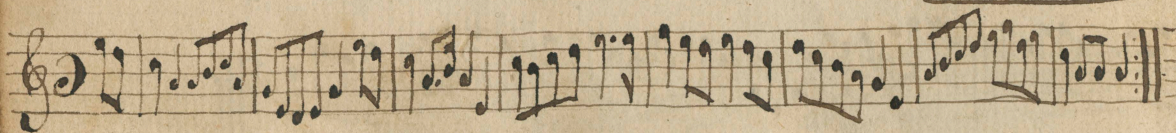


Durang's Horn pip



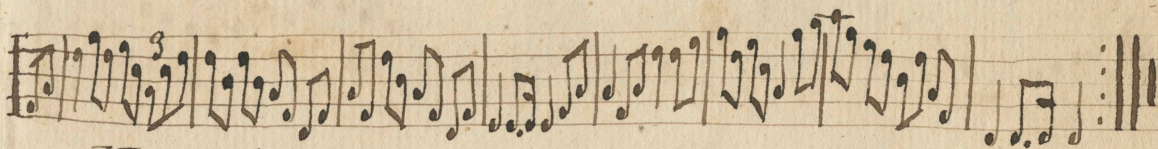
Eagle



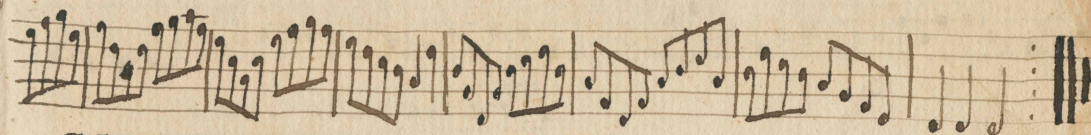
NorthamptonFishersDunklinCollegeBonny Lads ofSoldiersLogan

Matralls.

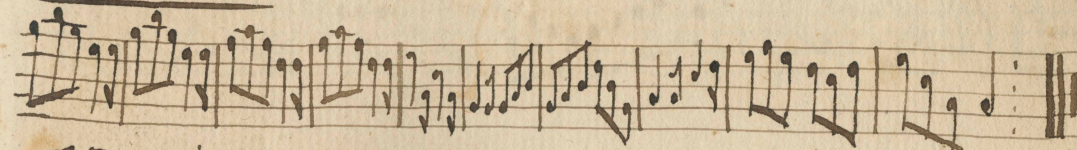
31



Hornpipe.



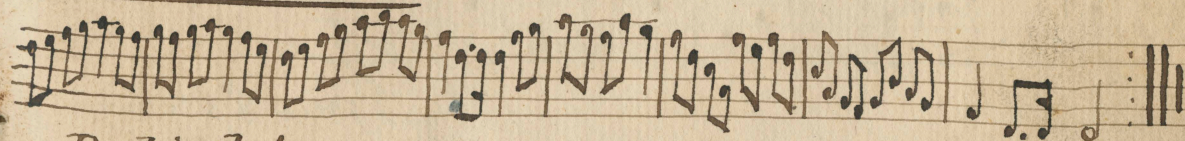
House.



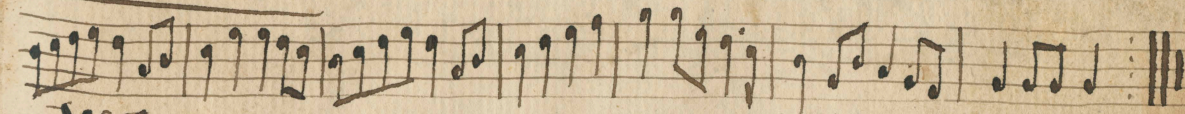
Hornpipe.



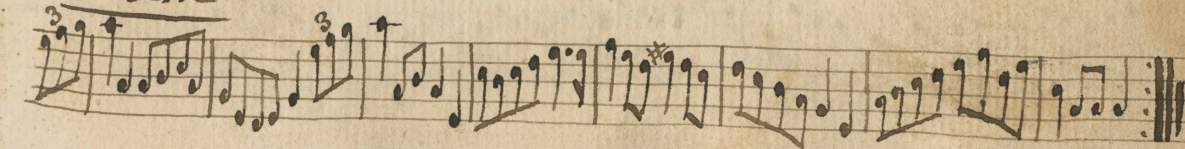
Aberdeen.

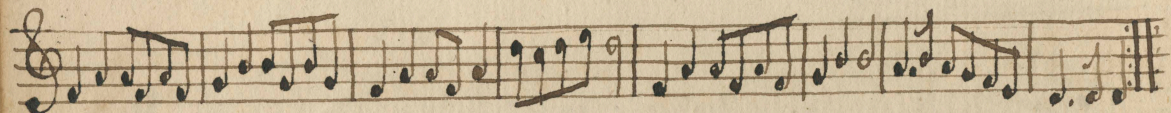
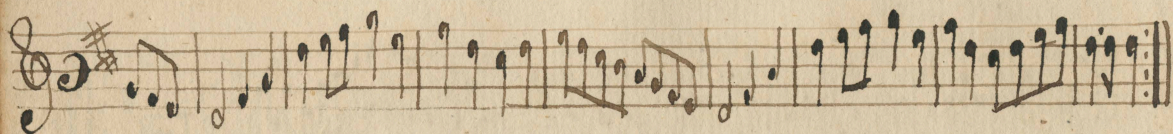
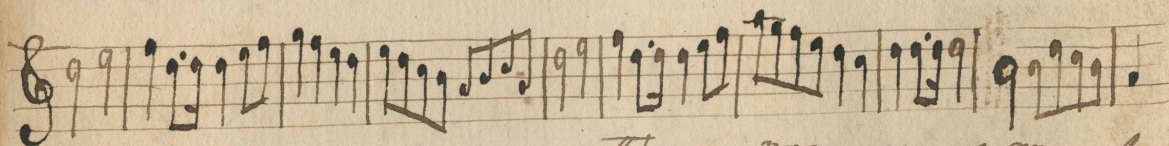
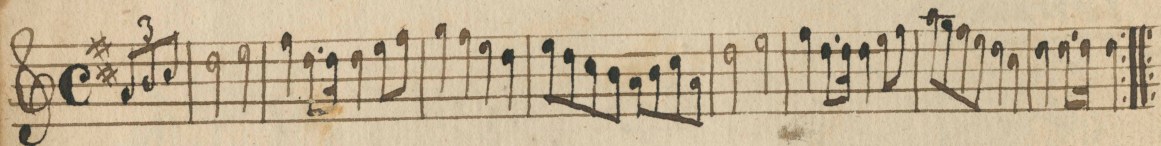
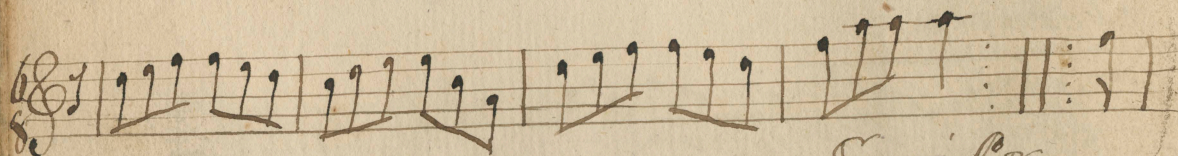
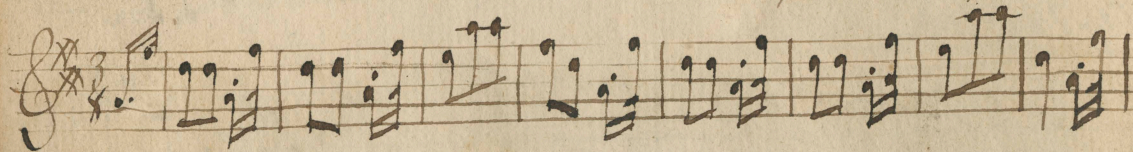


Delight.



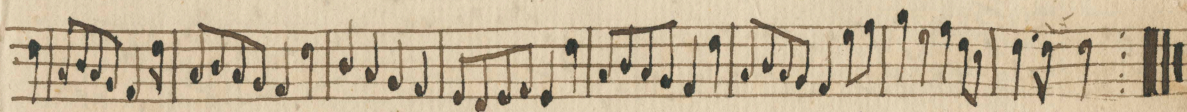
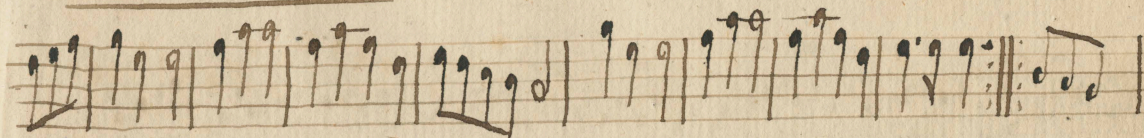
Water.



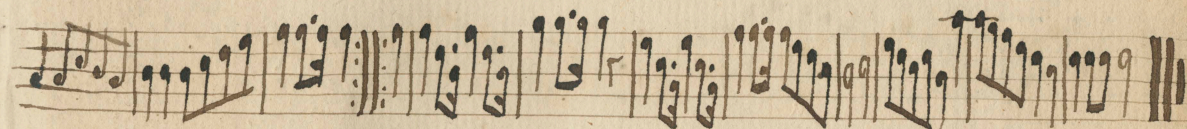
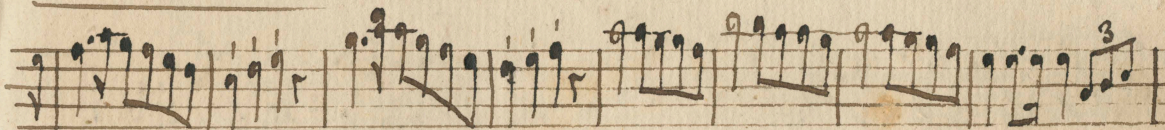
YorkFrench NationalFree Masons' MarchSwiss

Fusilleer.

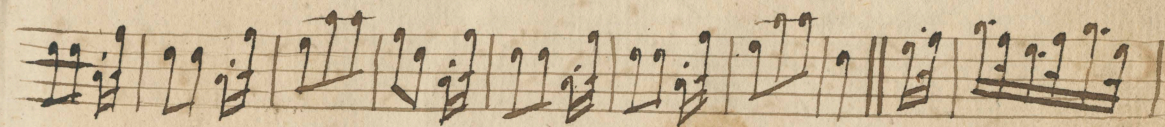
33

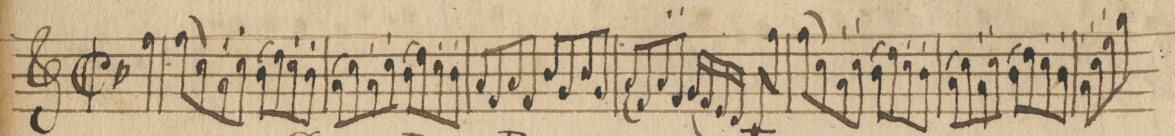


March.

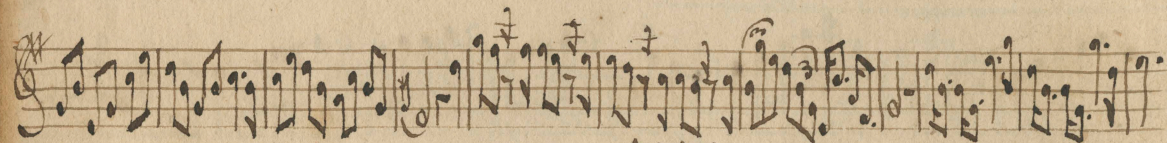
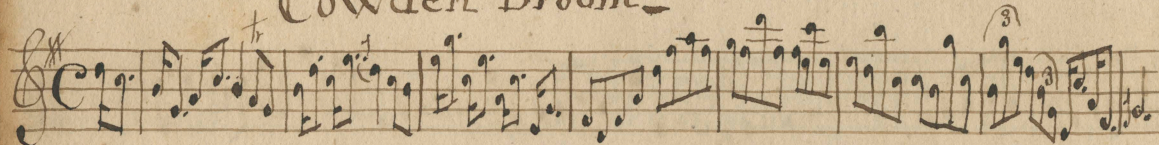


Waltz.

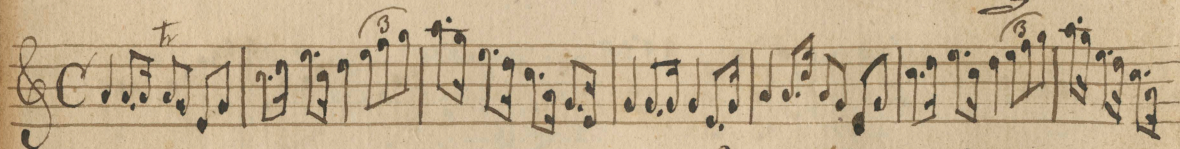




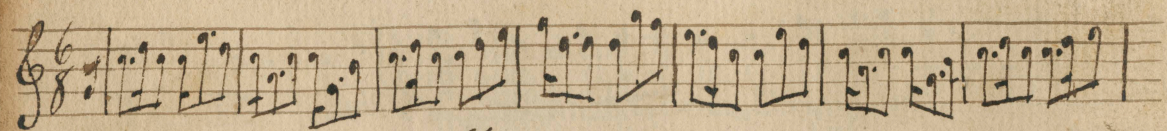
Cowden Broom



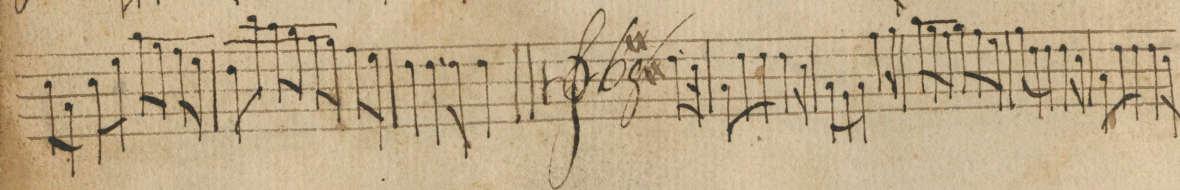
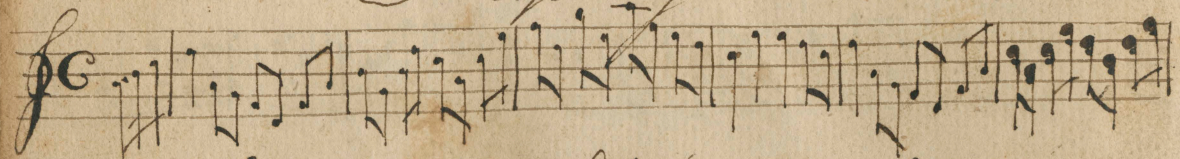
The Blazing



Cathleen Mc Cree



Murphy's Rant

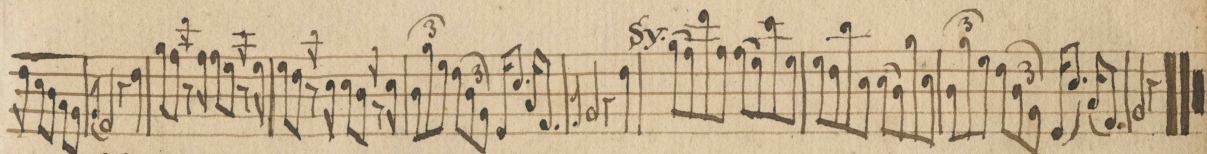
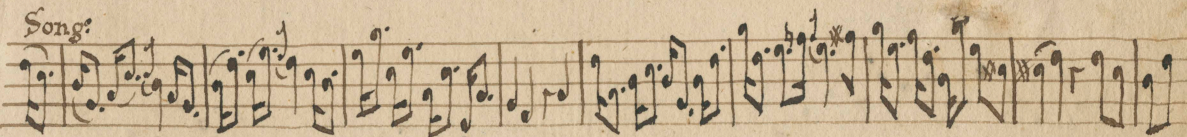


Hornpipe

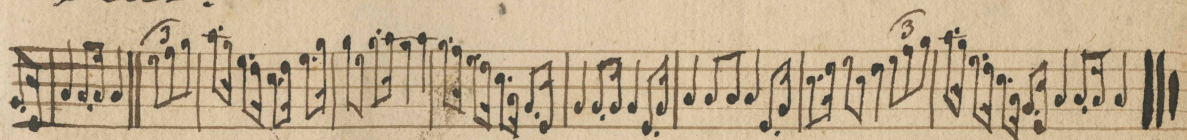
35



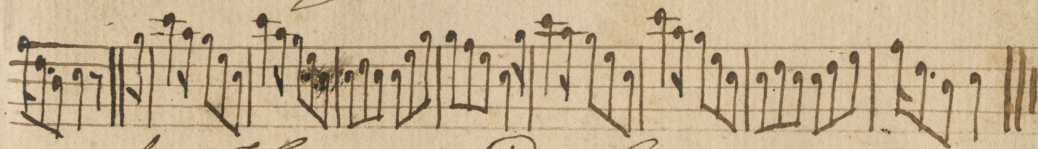
Song:



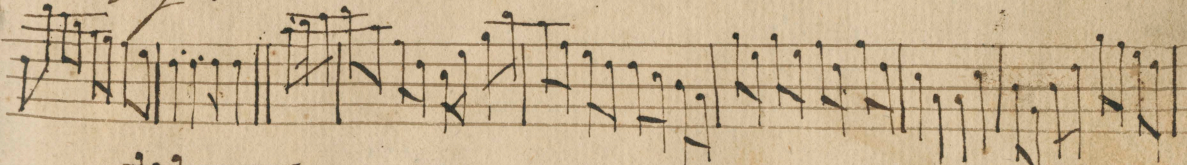
Star:

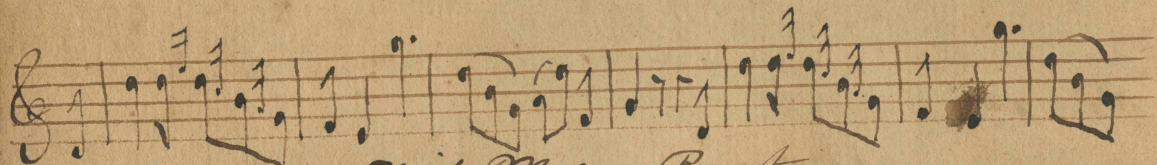
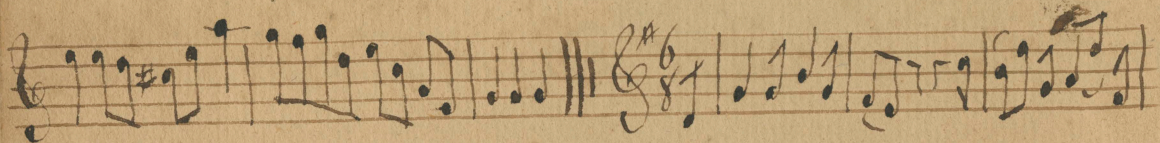


Syracuse L. Crosby



by Horace Graham



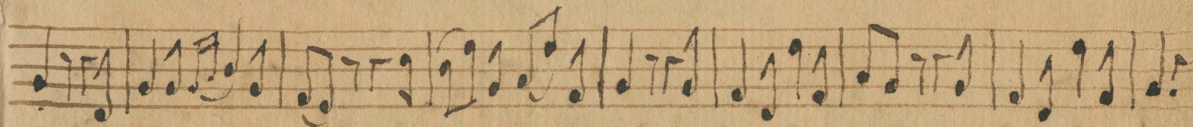
*Maid**Miss More's Rant*

Hornpipe

37



of Lodi



Trip to Holland ~

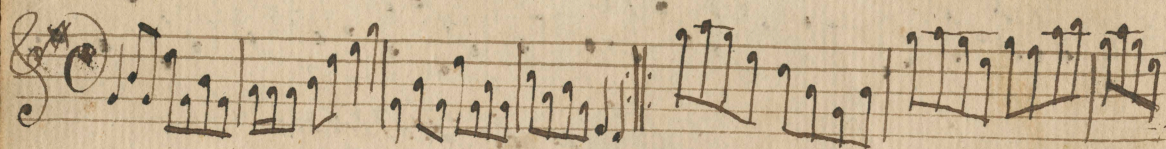


Well done Jack



The 12th of March by Whitlock

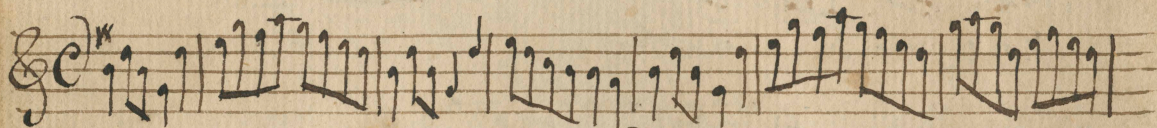
The Peep of Day



Chester Castle



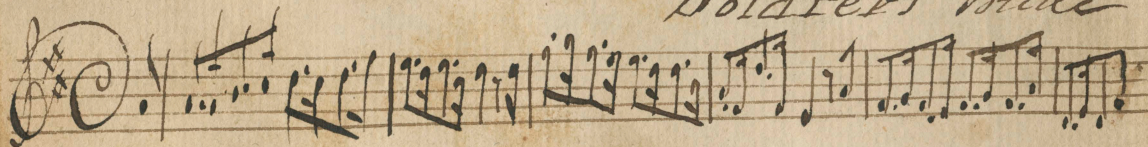
Butcher's Round

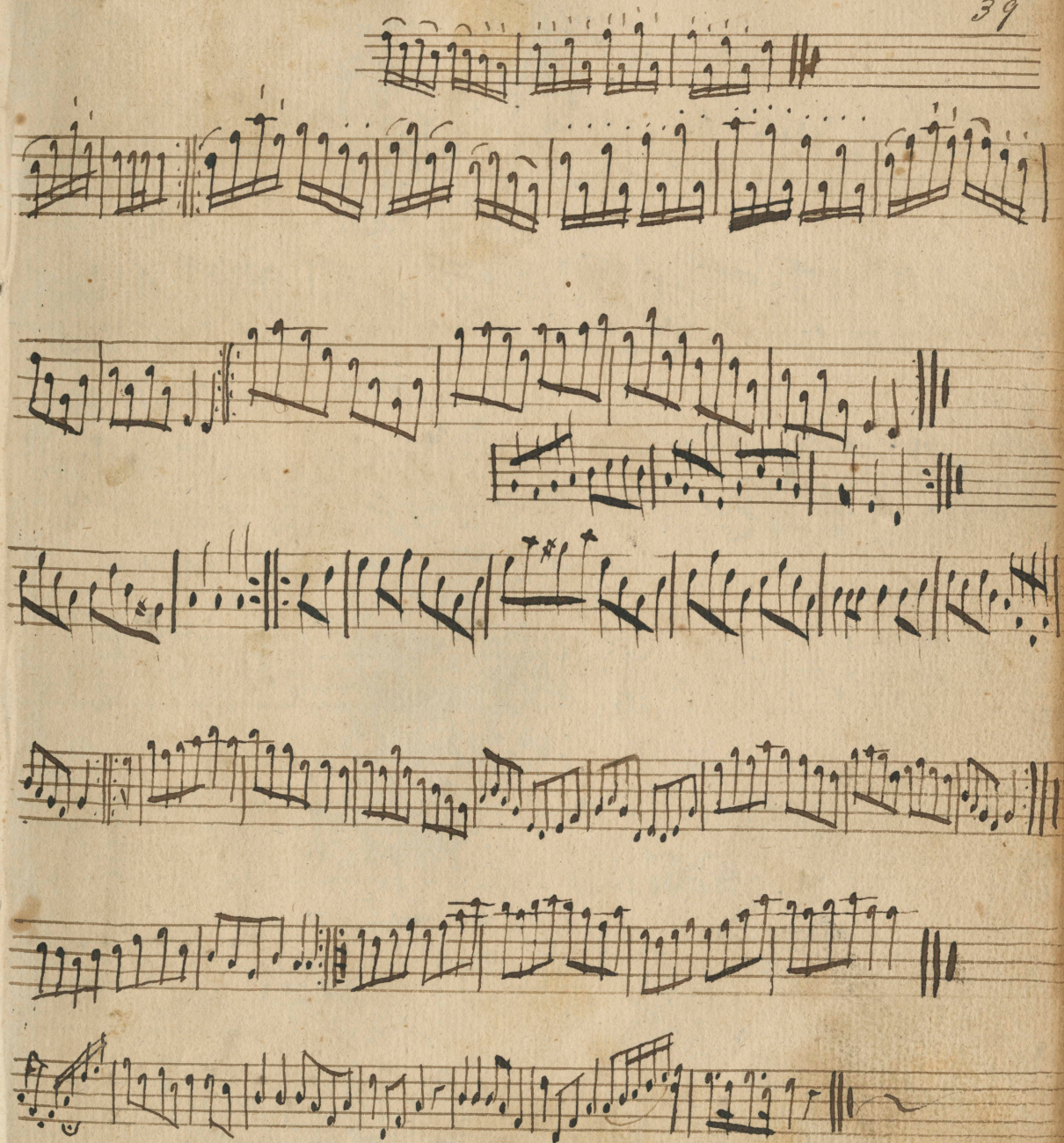


Carandaigne's Assembly

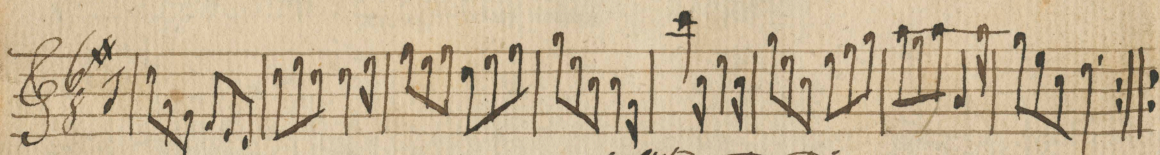


Soldiers' Bride

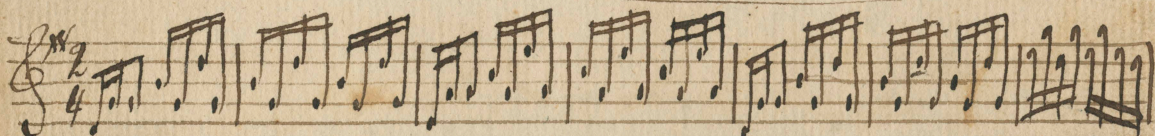




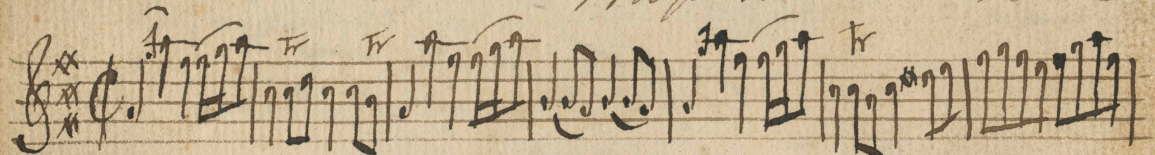
Take care of your cap-



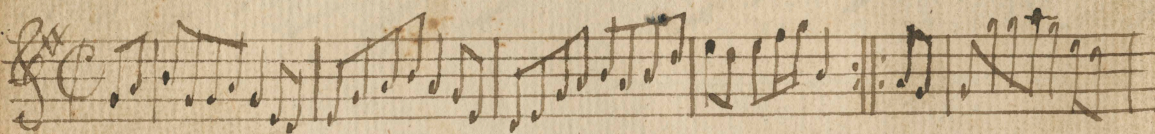
The Robin



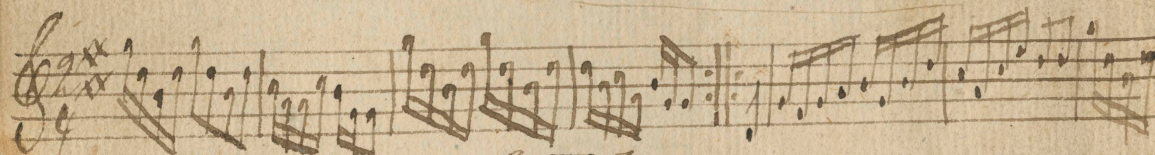
Miss M. Cloud's Reel



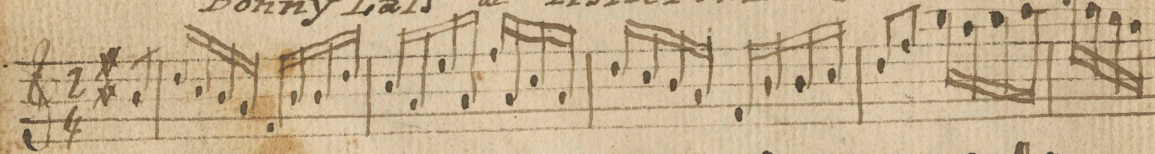
Long Hills of Morant.



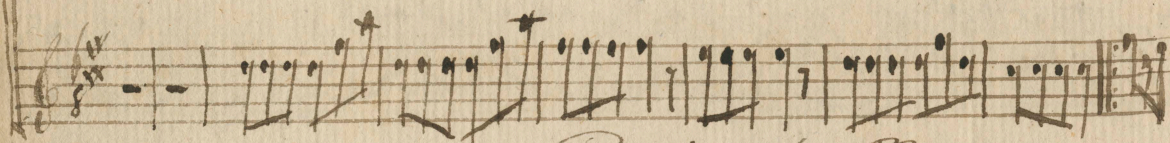
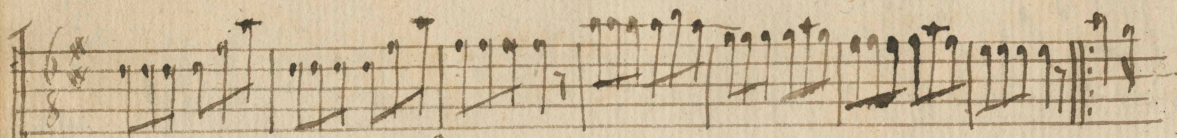
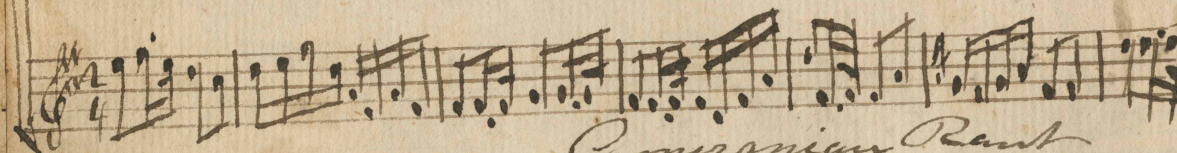
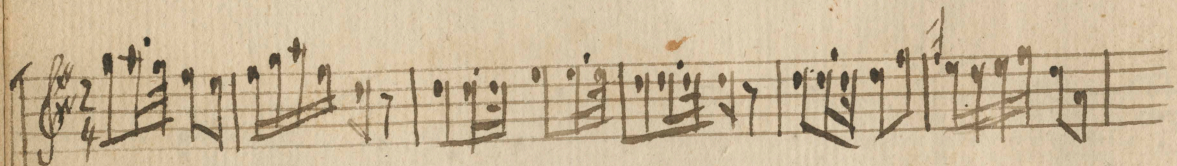
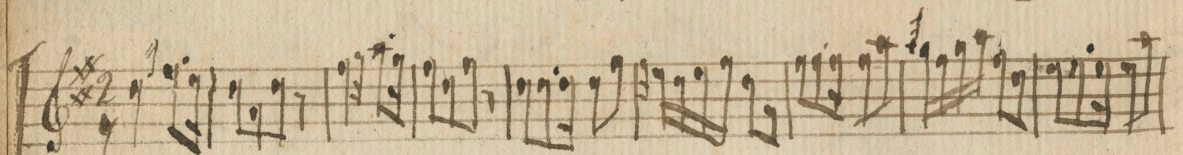
Ricketts Reel.



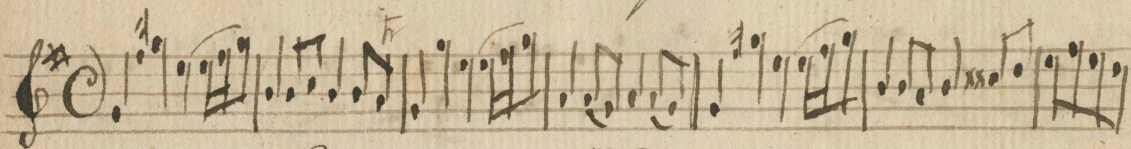
Bonny Lads of Fisherow



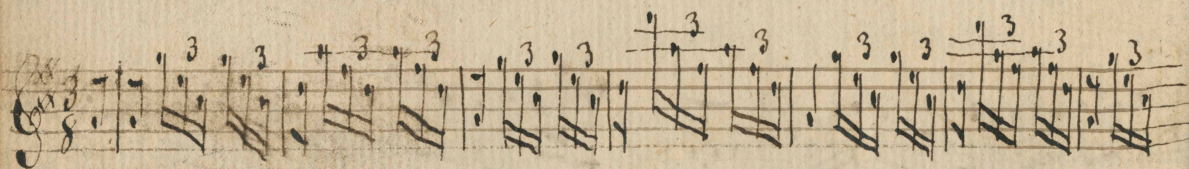
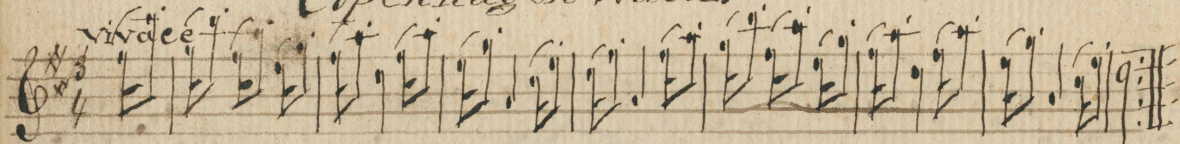


The Wood Cutters*President's March**Cameroonian Rant*



Miss M^c Clouds.

Copenhagen Waltz.

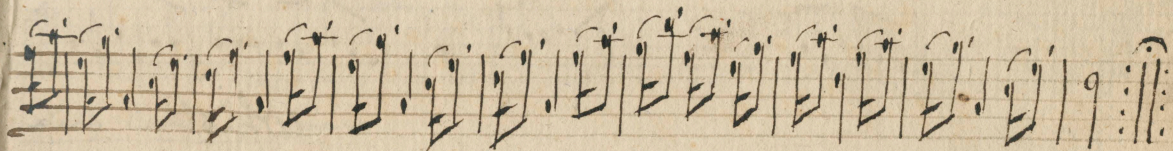
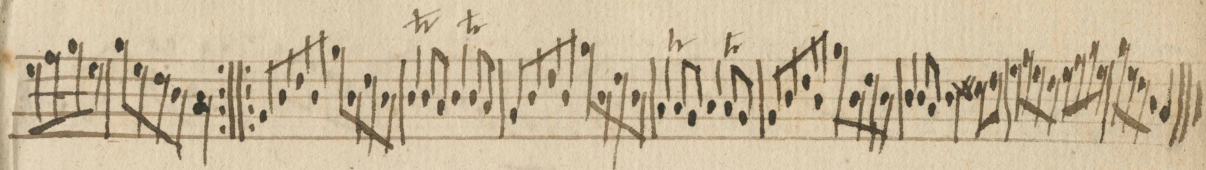


Philadelphia

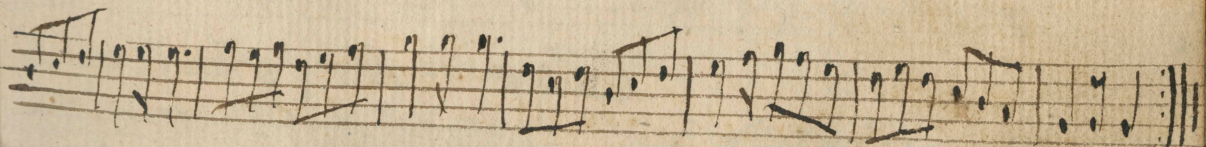
Chelmsford Races.

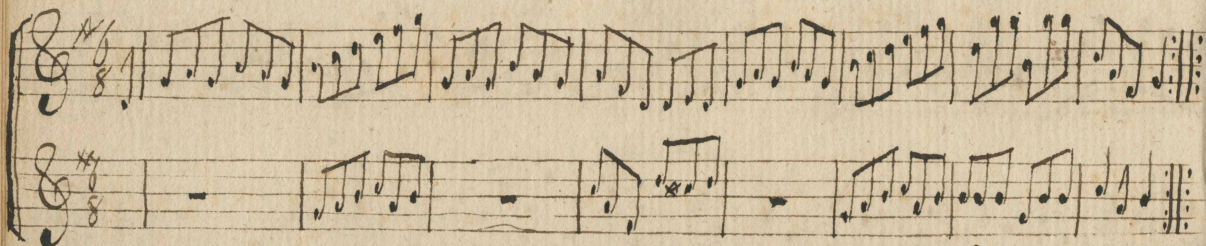
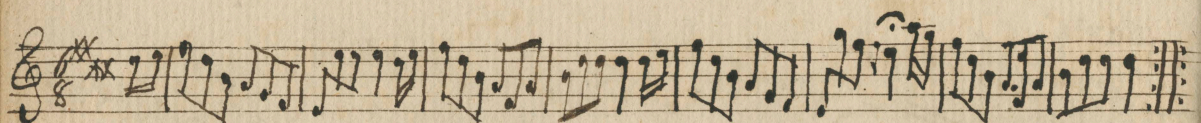
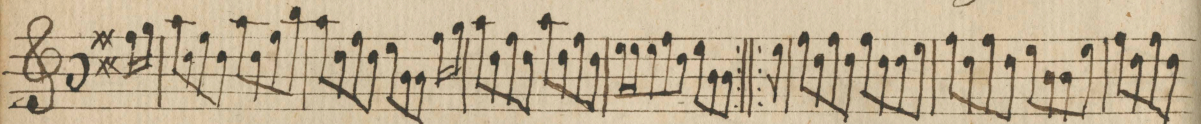
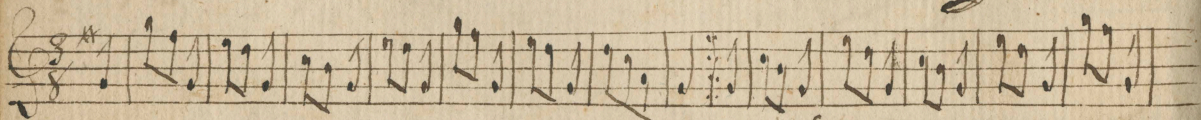
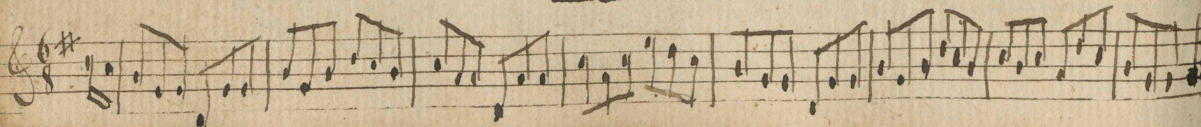
Reel, or Mr. Leeds

45



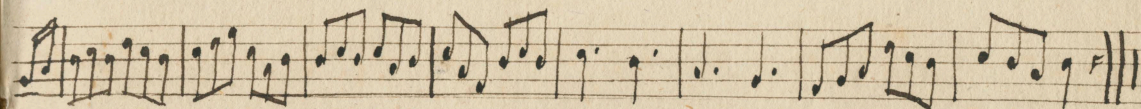
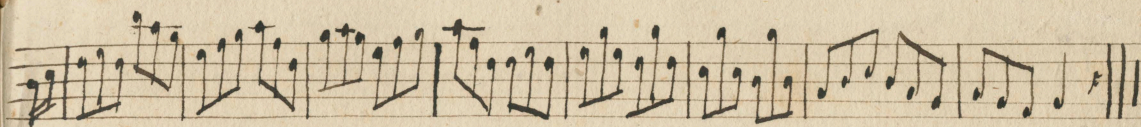
March



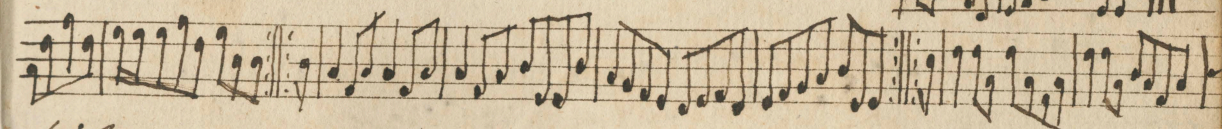
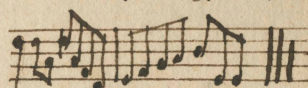
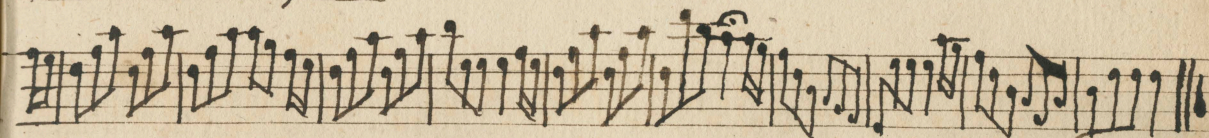
Capt.*Whistle and**Grigs' Pipes**Ricketts - Horn**Hungarian**Sheela na**The Irish*

Barry's Whim

47



S. ll come to you



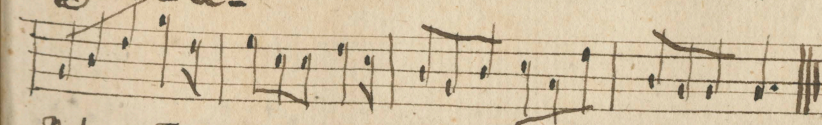
Wipe



Waltz.



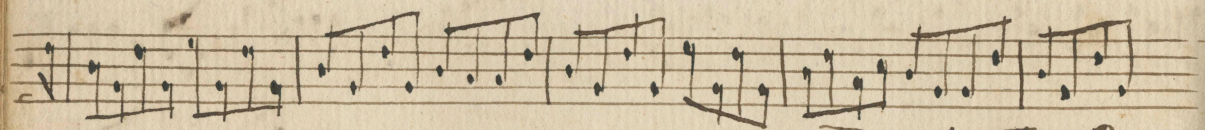
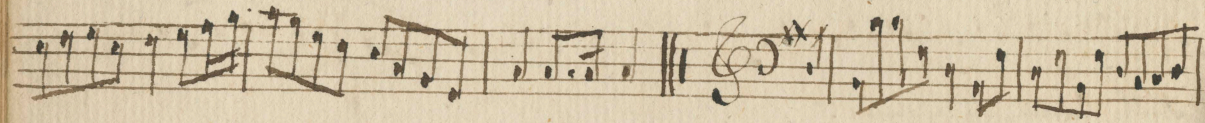
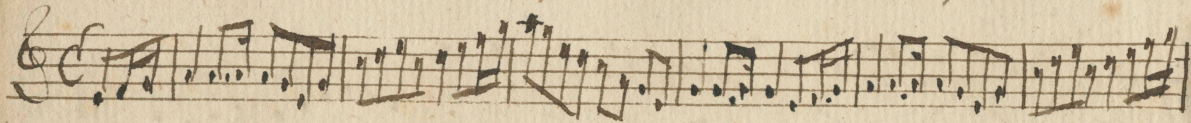
Guitar.



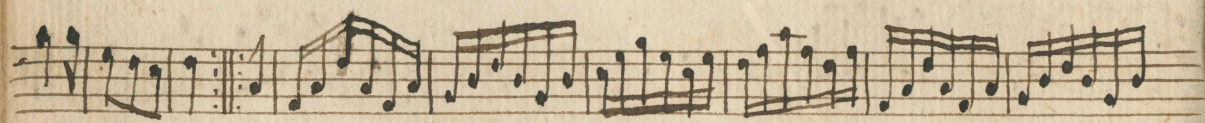
Waltz or can.



Quona parte



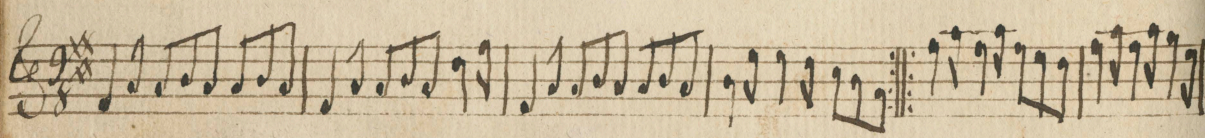
Dutchels of



The Cottage

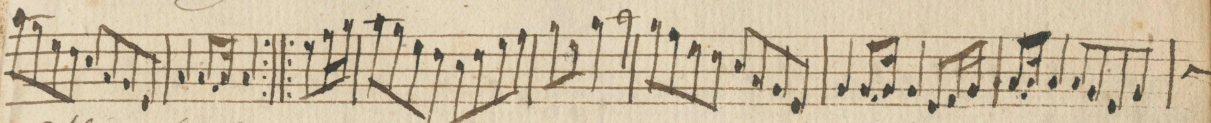


Pluett's

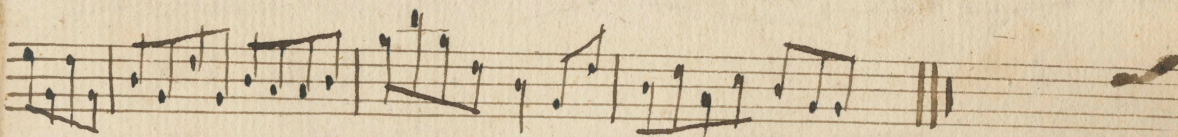
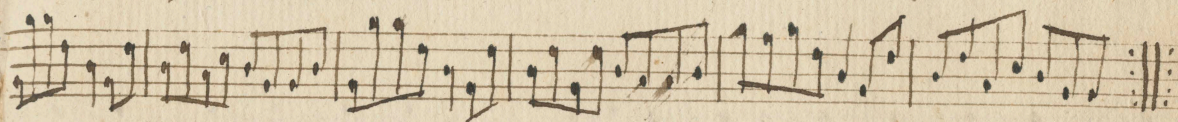


Crooping the Rhine

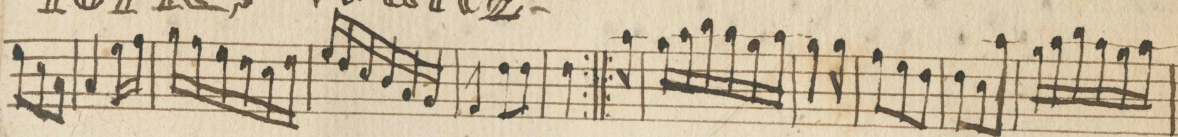
49



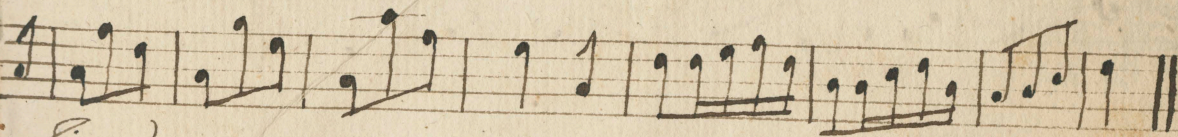
The Lass in you Town



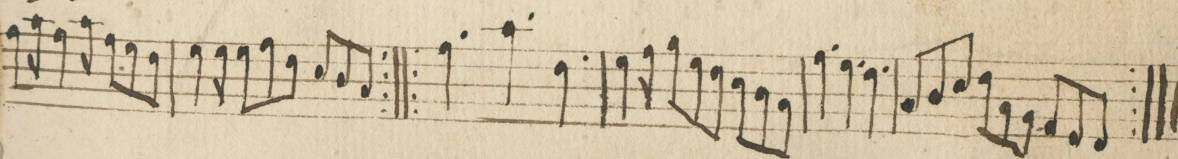
York's Waltz



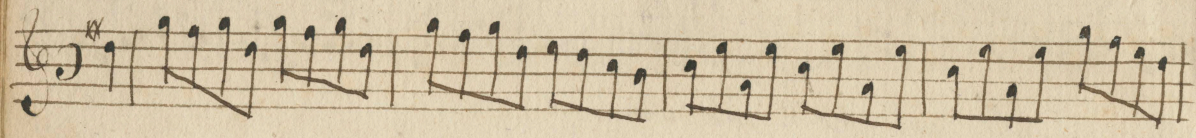
Waltz.



Sigg



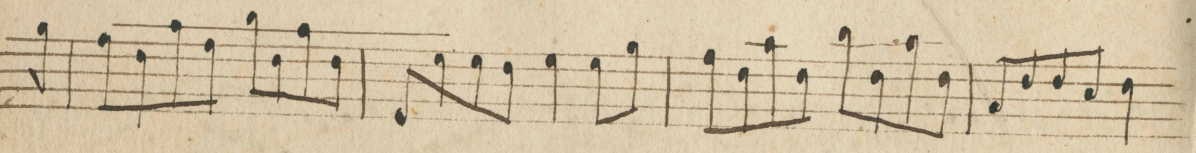
The ID—L



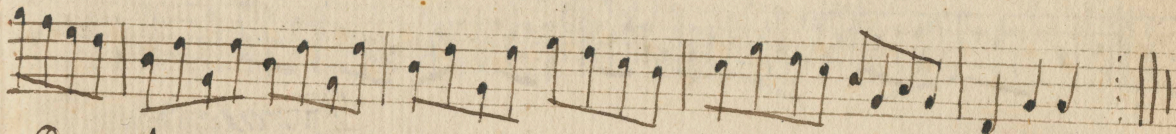
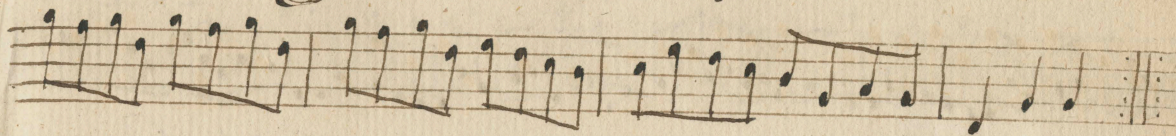
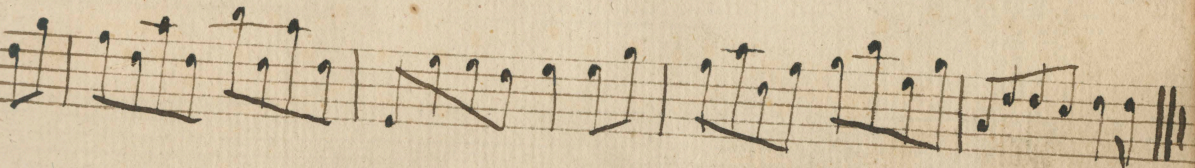
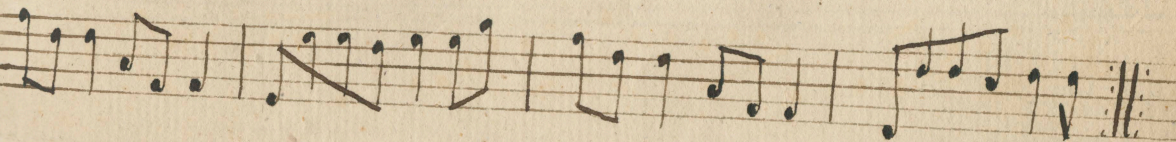
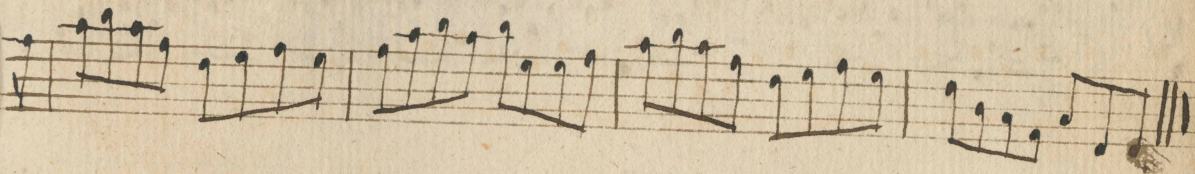
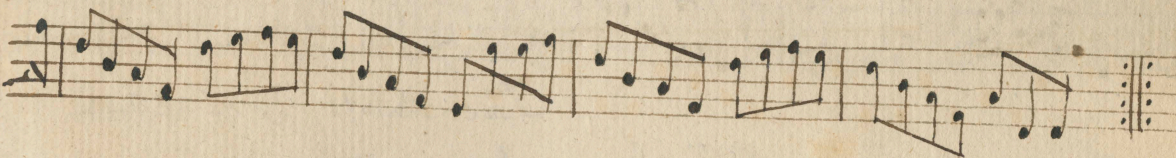
Lady Lucy



Life Hunt Reel



among the Tailors

Campbell's Reel

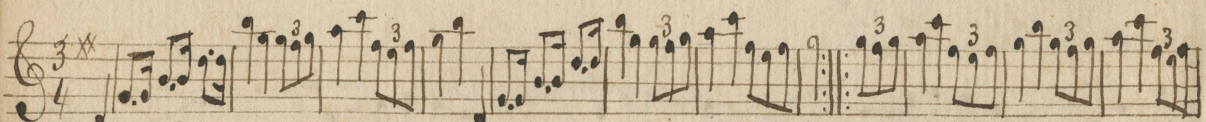
New Century



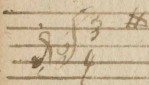
Paddy Carey



Tyrollese



The Anti Tyrolean



Hornpipe

53



Waltz.



Waltz.



88

Reduce $\sqrt{x-a} = \sqrt{x} - \frac{1}{2}\sqrt{a}$. Squaring both sides -

29

Ex. 10.

$$\begin{aligned} & \sqrt{x-a} = \sqrt{x} - \frac{1}{2}\sqrt{a} \\ & \sqrt{x-a} + \frac{1}{2}\sqrt{a} = \sqrt{x} \\ & x-a = x - \sqrt{ax} + \frac{a}{4} \quad \text{Transposing } x - \sqrt{ax} + \frac{a}{4} \\ & \sqrt{ax} = a + \frac{a}{4} \\ & 4\sqrt{ax} = 4a + a = 5a \quad \text{Squaring both sides} \\ & 16ax = 25a^2 \\ & x = \frac{25a^2}{16a} = \frac{25a}{16} \end{aligned}$$

11 Ex. Page 129 Day's Algebra. Reduce $\sqrt{5x} \sqrt{x+2} = 2 + \sqrt{5x}$. Squaring

Ex. 11.

$$\begin{aligned} & \sqrt{5x} \sqrt{x+2} = 2 + \sqrt{5x} \quad \text{both sides} \\ & 5x+10 = 4 + 2\sqrt{5x} + 5x \\ & 5x+10 = 4 + 2\sqrt{5x} + 5x \quad \text{Transf.} \\ & -5x-4 = 2\sqrt{5x} \quad \text{Squaring again} \end{aligned}$$

12. Reduce $x-ax = \sqrt{x}$

$$\begin{aligned} 1) & x^2 - ax^2 = x \quad \text{Clear of fractions} \\ & \quad \quad \quad \text{Dividing by } x \\ 2) & x - ax = 1 \quad \text{Dividing again by } x \\ & 1-a = \frac{1}{x} \quad \text{This } \frac{1}{x} \text{ is the reciprocal} \\ & \text{of } x; \text{ and hence, if } 1-a = \frac{1}{x} \text{ the reciprocal} \\ & \text{of } x \text{ then must } x = \frac{1}{1-a} \text{ the reciprocal of } 1-a. \end{aligned}$$

Ex. 14. Reduce $\sqrt{x} + \sqrt{a+x} = 2a$

$$\begin{aligned} & \sqrt{a+x} = 2a - \sqrt{x} \\ & \sqrt{a+x} + \sqrt{x} = 2a \quad \text{Squaring both sides} \\ & a+x = 4a^2 - 4a\sqrt{x} + x \\ & a = 4a^2 - 4a\sqrt{x} \\ & 4a\sqrt{x} = 4a^2 - a \\ & x = \frac{4a^2 - a}{4a} = \frac{a(4a-1)}{4a} = \frac{4a-1}{4} \quad \text{Ans.} \end{aligned}$$

Ex. 15. Reduce $x + \sqrt{a^2+x^2} = 2a^2$

$$\begin{aligned} & \sqrt{a^2+x^2} = 2a^2 - x \\ & \sqrt{a^2+x^2} + x = 2a^2 \quad \text{Squaring both sides} \\ & a^2+x^2 = 4a^4 - 4a^2x + x^2 \\ & a^2 = 4a^4 - 4a^2x \\ & 4a^2x = 4a^4 - a^2 \\ & x = \frac{4a^4 - a^2}{4a^2} = \frac{a^2(4a^2-1)}{4a^2} = \frac{4a^2-1}{4} \quad \text{Ans.} \end{aligned}$$

Ex. 16. Reduce $x+a = \sqrt{a^2+x^2} + \sqrt{b^2+x^2}$ Squaring both sides

$$\begin{aligned} 2) & x^2 + 2ax + a^2 = a^2 + x^2 + b^2 + x^2 + 2bx + b^2 \\ & x^2 + 2ax + a^2 = 2x^2 + b^2 + 2bx + 2a^2 \\ & x^2 + 4ax = b^2 + x^2 \quad \text{Transposing } a^2 + x^2 \\ & 4ax = b^2 \\ & x = \frac{b^2}{4a} \quad \text{Ans.} \end{aligned}$$

Ex. 17. Reduce $\sqrt{2+x} + x = \frac{4}{x}$

$$\begin{aligned} & \sqrt{2+x} + x = \frac{4}{x} \\ & \sqrt{2+x} = \frac{4}{x} - x \\ & \sqrt{2+x} + x = \frac{4}{x} \quad \text{Squaring both sides} \\ & 2+x = \frac{16}{x^2} - 2 + 2\sqrt{2+x} + x^2 \\ & 2+x = \frac{16}{x^2} - 2 + 2\sqrt{2+x} + x^2 \\ & 4+x = \frac{16}{x^2} + 2\sqrt{2+x} + x^2 \\ & 4+x = \frac{16}{x^2} + 2\sqrt{2+x} + x^2 \\ & x = \frac{4}{x} - x \end{aligned}$$

60 18th Ex. Reduce $\sqrt{x-32} = 16-\sqrt{x}$. Squaring both sides.

$$x-32 = 256 - 32\sqrt{x} + x.$$

Transf. & uniting

$$32\sqrt{x} = 288$$

Dividing by 32

$$\sqrt{x} = 9$$

$$x = 81.$$

~~Square both Members~~

Ex 19. Reduce $\sqrt{4x+17} = 2\sqrt{x}+1$

Square both sides

Transf. $4x+1$

$$4x+17 = 4x+4\sqrt{x}+1$$

$$4x-1$$

$2\sqrt{x}+1$

$$4\sqrt{x}+4\sqrt{x}+1$$

1st term = $4\sqrt{x}$

$$4\sqrt{x} = 16$$

$$\sqrt{x} = \frac{16}{4}$$

$$x = 4^2 = 16.$$

Ex. 20. Reduce $\sqrt{6x-2} = 4\sqrt{6x-9}$

Clear fractions

$$\sqrt{6x+2} = 4\sqrt{6x+6}$$

$$4\sqrt{6x-9}$$

$$4\sqrt{36x^2+8\sqrt{6x}-18}$$

$$4\sqrt{36x^2+6\sqrt{6x}-12}$$

$$4\sqrt{36x^2+8\sqrt{6x}-18} = 4\sqrt{36x^2+6\sqrt{6x}-12}$$

Cancelling $4\sqrt{36x^2}$ on both sides

$$-8\sqrt{6x}-18 = -6\sqrt{6x}-12$$

Transf. all the signs & transf. &

$$6 = \sqrt{6x}$$

Squaring both members

$$36 = 6x : x = \frac{36}{6} = 6.$$

Prob. 8 - pag. 134. What 2 numbers are those whose Diff. is to the greater as 2:9 and the difference of whose squares is 128. Let $2x =$ their diff. & $9x =$ the greater number.

Then will $7x =$ the lesser - But the product of the sum & difference of any two numbers is equal to the diff. of their squares. hence $(9x+7x) = 9x+7$

Proof. Substituting 2 in the room of x -

$$9 \times 2 = 18$$

$$7 \times 2 = 14$$

Numbers are 32 & 14

$$(9x-7x) = 9x-7$$

$$81x^2 - 49x^2 = 49x^2 - 63x^2 = 49x^2$$

$$81x^2 - 49x^2 = 128$$

$$32x^2 = 128$$

$$x = \frac{128}{32} = 4$$

$$x = \sqrt{4} = 2.$$

The product of any number multiplied by 4, is equal to the square of twice the square root of such number. Thus $49 \times 4 = 196$. and twice the $\sqrt{49} = 14^2 = 196$.

Prob 12 Two Travellers, A & B set out to meet each other - A leaving the Town of C. at the same time & B left the Town of D. They travelled the direct road between C & D; and on meeting, it appeared that A had travelled 18 miles more than B, and that A could have gone B's distance in 15 days $\frac{3}{4}$ - but B would have been 28 days in going A's distance -

Required the distance between the Towns.

Let x = the miles A travelled

then $x - 18$ = the miles B travel $\frac{x - 18}{15\frac{3}{4}}$ = a day's travel of A

$\frac{x}{28}$ = B's daily travel

$$\text{Since } x : x - 18 :: \frac{x - 18}{15\frac{3}{4}} : \frac{x}{28}$$

$$\frac{x^2}{28} = \frac{x^2 - 36x + 324}{15\frac{3}{4}} \text{ Multiplying this equation by } 4 \text{ to strike } \frac{4}{4}$$

$$\frac{4x^2}{112} = \frac{4x^2 - 144x + 1296}{63} \text{ Clearing this of fractions}$$

$$25) 252x^2 = 448x^2 - 16128x + 145152. \text{ Dividing by } 28$$

$$9x^2 = 16x^2 - 576x + 5184 \text{ Transf. & uniting terms}$$

$$7x^2 - 576 = -5184 \text{ Completing the square by multiplying the equation by } 4 \text{ times the Co. of } x^2 \text{ of the highest power}$$

$$\frac{196x^2 - 16128x + 331776}{186624} = \frac{-145152 + 331776}{186624} \text{ The square of the Co. of } x \text{ is } 196 \text{ by adding both members}$$

$$\text{have } 14x^2 = 576 + 432$$

$$\frac{14 \overline{) 1008} 72 = x \text{ the distance A travelled, } \frac{54}{28} = \text{the distance B travelled}$$

So that 126 miles is the distance between C & D.

Another rule for Completing the square of a binomial.

If the highest power of the unknown quantity have a coefficient equal to only ONE multiply the equation by 4 - & add to both sides the square of the coefficient of the lowest power - if the Co. of the highest be 2-3-4 &c. Multiply the equation by 8-12-16 &c. & add the square of the Co. of the first power as before. The root extracted will leave the unknown quantity simply multiplied

Ex. 7. Eq 147 Reduce $\frac{x+4}{3} - \frac{7-x}{x-3} = \frac{4x+7}{9} - 1$ Clear fractions 9

$9x^2 + 9x - 108 + 27x - 189 = 12x^2 - 15x - 63$ Invert & simplify

$3x^2 - 78x = -315$ Divide by 3

$x^2 - 26x = -105$ Comp. Squ by add $\frac{1}{2}$ Co. of x

$x^2 - 26x + 169 = 169 - 105 = 64$

$x = 13 \pm \sqrt{64} = +8 = 21$

or $13 - 8 = -5$

8th Ex. Reduce $\frac{x^3 - 10x^2 + 1}{x^2 - 6x + 9} = x - 3$ Clearing fractions

$x^3 - 9x^2 + 27x - 27 = x^3 - 10x^2 + 1$ x^3 is balanced

$x^2 + 27x = 28$ Trans & unity terms

$4x^2 + 108x + 729 = 112 + 729 = 841$

$2x + 27 = \sqrt{841} = 29$

$2x = -27 + 29 = 2$ or $29 + 27 = 56 = 28$
 $x = 29 - 27 = 2 = 1$ Exty root

Ex 3 Reduce $4x - \frac{14-x}{x+1} = 14 = \frac{9}{4} 4x^2 + 4x - 14 + x = 14x + 14$

$x^2 - 9x = 7$

Comp. Squar $x^2 - 9x + \frac{81}{4} = 7 + \frac{81}{4} = \frac{529}{4}$

$x = \frac{9}{2} \pm \sqrt{\frac{529}{4}} = \frac{23+9}{2} = \frac{32}{2} = +4$

or $\frac{23-9}{2} = \frac{14}{2} = 7$ or $7\frac{1}{4}$

Ex 14 Reduce

2) $2\sqrt{x^2} + 3\sqrt{x} = 2$ - divide by 2

$x^2 + 3\sqrt{x} + \frac{9}{16} = 1 + \frac{9}{16}$ Comp. Squar

$x^2 = -\frac{3}{4} + \sqrt{1 + \frac{9}{16}}$ Exty root

$x = \left(-\frac{3}{4} + \sqrt{1 + \frac{9}{16}}\right)^2$ Involving both sides

The value of the quantity under the radical sign is $\sqrt{\frac{25}{16}} = \frac{5}{4}$; And the sum of $\frac{5}{4} - \frac{3}{4}$ is $\frac{2}{4}$, the Cube of which is $\frac{8}{64}$ or $\frac{1}{8}$ the value of x

64 Ex 10 Reduce $\frac{3x}{x+2} - \frac{x-1}{6} = x-9$ Clearing fractions
 $18x - x^2 - x + 2 = 6x^2 - 42x - 108$ Transf & uniting
 $7x^2 - 59x = 110$
 $196x^2 - 1652x + 3481 = 3080 + 3481$ Comp squ by mult by 28
 $14x = 59 \pm \sqrt{6561} = 81$ Ext root
 $x = 140 \div 14 = 10$ Ans.

Ex 11. Reduce $\frac{x}{a} + \frac{a}{x} = \frac{2}{a}$ Clear fractions
 a) $ax^2 + a^3 = 2ax$ Dividing by a
 $x^2 + a^2 = 2x$ Transf
 $x^2 - 2x = -a^2$ Comp square
 $x^2 - 2x + 1 = 1 - a^2$ Ext root
 $x = 1 \pm \sqrt{1 - a^2}$

Ex 9. Reduce $\frac{6}{x+1} + \frac{2}{x} = 3$ Clear front
 $6x + 2x + 2 = 3x^2 + 3x$ Trans & unit
 $3x^2 - 5x = -2$ mult by 12 & add square
 $36x^2 - 60x + 25 = 24 + 25 = 49$ of comp
 $6x = 5 \pm \sqrt{49} = 7$ Ext. root
 $x = \frac{7}{12} \div 6 = 2$ Ans.

Ex 18. Reduce $3x^2 - 2x^2 = 8$
 $36x^2 - 24x + 4 = 96 + 4$ Comp square
 $6x = 2 \pm \sqrt{96 + 4} = 10$ Ext. root
 $6x = 12$
 $x = \sqrt{12} \div 6 = 2$
 19. Omit the multiplication - transfer & change all the signs - and

Ex. 19. Reduce $2(1+x-x^2) - \sqrt{1+x-x^2} = -\frac{1}{2}$
 $x^2 - x + \frac{1}{4} = (\sqrt{x^2 - x + 1}) = 1 + \frac{1}{4}$ complete square.
 $x = \frac{1}{2} + \sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \frac{1}{2} \pm \sqrt{49}$ Ext. root. $1 + \frac{1}{4} = \frac{5}{4}$ $\frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$
 But this is not right.

See page 67.

Ex 20. $\sqrt[3]{x^3 - a^3} = x - b$ Cube both sides

$$\begin{array}{r} x^3 - b^3 \\ -bx^2 + b^2x \\ \hline x^3 - 2bx^2 + b^2x \\ -bx^2 + 2b^2x - b^3 \\ \hline \end{array}$$

$x^3 + a^3 = x^3 - 3bx^2 + 3b^2x + b^3$ Express the x cubes

$3b) 3bx^2 - 3b^2x = a^3 - b^3$ Change all the signs transfer $a^3 + b^3$ and

$$x^2 - bx = \frac{a^3 - b^3}{3b}$$

Divide by $3b$
 $x^2 - bx + \frac{b^2}{4} = \frac{a^3 - b^3}{36} + \frac{b^2}{4}$ Comp Square

Extract
 $x = \frac{b}{2} + \sqrt{\frac{a^3 - b^3}{36} + \frac{b^2}{4}}$
 $\frac{\sqrt{4a^3 - 4b^3 + 36b^3}}{12b} = \frac{4a^3 - b^3}{12b}$

Ex. 21. Reduce $\frac{\sqrt{4x+2}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}}$

$$\frac{\sqrt{4x+2}}{\sqrt{x}}$$

$$\left(\frac{4-\sqrt{x}}{4+\sqrt{x}} \right)$$

$$\sqrt{4x+2} = 2\sqrt{x} + \sqrt{2x} = 16 - x \text{ transfer}$$

$$\frac{16 - 4\sqrt{x}}{4 + \sqrt{x}}$$

$$\frac{3x + \sqrt{2x} = 16 - x \text{ sq both sides}}{3x + \sqrt{2x}}$$

$$\frac{16 - 4\sqrt{x} + 4\sqrt{x} + x}{16 - x}$$

$$\frac{9x^2 + 3x\sqrt{2x}}{9x^2 + 3x\sqrt{2x}}$$

$$\frac{+3x\sqrt{2x} + 2x}{+3x\sqrt{2x} + 2x}$$

$$\frac{9x^2 + 6x\sqrt{2x} + 2x = 256 \text{ Extract root.}}{9x^2 + 6x\sqrt{2x} + 2x = 256}$$

$$\frac{3x + x = 16}{3x + x = 16}$$

$$\frac{4x = 16}{4x = 16}$$

$$\frac{x = 16 \div 4 = 4}{x = 16 \div 4 = 4}$$

Not quite satisfactory

21st Ex Again

Ex 13. Reduce $\frac{x^4 - x^2}{2} = -\frac{1}{32}$

$$128x^4 - 64x^2 = -8$$

$$x^4 - \frac{1}{2}x^2 = -\frac{1}{16}$$

$$x^4 - \frac{1}{2}x^2 + \frac{1}{16} = -\frac{1}{16} + \frac{1}{16}$$

$$x^2 = \frac{1}{4}$$

$$x = \sqrt{\frac{1}{4}}$$

Ex 21. again. $\frac{\sqrt{4x+2}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}}$

$$\frac{\sqrt{4x+2}}{\sqrt{x}} = \frac{4-\sqrt{x}}{4+\sqrt{x}}$$

$$\frac{2\sqrt{x} + \sqrt{2x}}{2\sqrt{x} + \sqrt{2x}} = \frac{16 - x}{16 - x}$$

$$\frac{9x^2 + 3x\sqrt{2x}}{9x^2 + 3x\sqrt{2x}} = \frac{256}{256}$$

$$\frac{3x + x = 16}{4x = 16}$$

$$x = 16 \div 4 = 4$$

$$\sqrt{4x+2}/x = 16 - x \quad \sqrt{4x} = 2\sqrt{x}$$

$$2x + 2\sqrt{x} = 16 - x \text{ Transfer } 2x \text{ to right}$$

$$2\sqrt{x} = 16 - 3x$$

$$4x = 256 - 96x + 9x^2 \text{ Trans term}$$

$$9x^2 - 100x = -256 \text{ Comp square}$$

$$\frac{324x^2 - 3600x + 10000 = 9216 + 10000 = 18400 \text{ Extract root}}{18x = 100 \pm \sqrt{564} = 28}$$

$$x = 100 - 28 = 72 \div 18 = 4$$

66 Ex 15. Reduce $\frac{1}{2}x - \frac{1}{3}\sqrt{x} = 22\frac{1}{2}$ Dividing the eqn. by $\frac{1}{2}$
 $x - \frac{2}{3}\sqrt{x} = 44\frac{1}{3} = 44\frac{1}{3}a^2$
 $x - \frac{2}{3}\sqrt{x} + \frac{4}{9} = 44\frac{1}{3} + \frac{4}{9}$ Comply square

Shorter method.
 $\frac{1}{2}x - \frac{1}{3}\sqrt{x} = 22\frac{1}{2}$ Clear fraction
 $18x - 12\sqrt{x} = 798$ Divd. by 6
 $3x - 2\sqrt{x} = 133$ Comp. eqn

$$36x - 24\sqrt{x} + 4 = 4 + 1596 = 1600$$

$$6\sqrt{x} = 2 \pm \sqrt{1600} = 40 \pm 2 = 42$$

$$\sqrt{x} = 42 \div 6 = 7$$

$$x = 7^2 = 49.$$

$$\sqrt{x} = \frac{2}{3} \pm \sqrt{44\frac{1}{3} + \frac{4}{9}}$$

$$x = \frac{2}{3} \pm \sqrt{44\frac{1}{3} + \frac{4}{9}}^2$$

$$44\frac{1}{3} = 44\frac{1}{3}$$

$$\frac{4}{9} = \frac{1}{9}$$

$$44\frac{1}{3} + \frac{1}{9} = 44\frac{4}{9}$$

$$\sqrt{44\frac{4}{9}} = \frac{40}{3} + \frac{2}{3}$$

$$\frac{40}{3} + \frac{2}{3} = \frac{42}{3} = 14$$

$$\frac{2}{3} \pm 14 = \frac{2}{3} + 14 = \frac{44}{3}$$

$$\frac{44}{3} = 14\frac{2}{3}$$

$$\frac{44}{3} + \frac{1}{9} = \frac{441}{9} + \frac{1}{9} = \frac{442}{9}$$

$$\sqrt{\frac{442}{9}} = \frac{\sqrt{442}}{3}$$

$$\frac{\sqrt{442}}{3} = \frac{21}{3} = 7$$

$$x = 49.$$

Ex. 16. Reduce $2x^2 - x^2 + 96 = 99$. Transf. 96 $99 - 96 = 3$
 $16x^2 - 8x^2 + 1 = 1 + 24$ Compl square by multy by 8 (radix) left
 $4x^2 = 1 \pm \sqrt{1 + 24} = 25 = 5$ Ex. root
 $x^2 = 1 + 5 = 6 = \frac{1}{4}$
 $x = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{2} = \frac{1}{2}\sqrt{6}.$

Ex 17 - Reduce $(10+x)^{\frac{1}{2}} - (10+x)^{\frac{1}{4}} = 2$. Reduce to a com. index - x
 $\sqrt{10+x} - \sqrt[4]{10+x} + \frac{1}{4} = 2 + \frac{1}{4}$ Comply square
 $\sqrt{10+x} = \frac{1}{4}\sqrt{2+\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$. Square both sides
 $10+x = 4$
 $x = 4 - 10 = -6$. Ans.

The $\frac{1}{4}$ root of $10+x$ subtracted from the square root of the same quantity, leaves the $\frac{1}{4}$ root of that quantity: The $\sqrt[4]{10+x} = 2$.
 $10+x$ therefore is equal to $2^4 = 16$ and $x = 16 - 10 = 6$. Answer

Ex. 18. Reduce $3x^{2n} - 2x^n = 8$
 $36x^{2n} - 24x^n + 4 = 36 + 4 = 100$
 $6x^n = 2 \pm \sqrt{100} = 10 \pm 2 = 12$ Ex. root
 $x^n = \frac{12}{6} = 2$

Ex. 22. Reduce $x^{\frac{4}{5}} + x^{\frac{3}{5}} = 756$

$$x^{\frac{4}{5}} + x^{\frac{3}{5}} + \frac{1}{4} = 756 + \frac{1}{4}$$

$$x^{\frac{3}{5}} = -\frac{1}{2} + \sqrt{756 + \frac{1}{4}} = 756 + \frac{1}{4}$$

Comp. 9 square

Ex^{ty} root

~~Involving~~ — $x^{\frac{2}{5}} = \sqrt[5]{27} = 3$

Involving $x^{\frac{2}{5}} = 9$

$$\frac{2 \times 3}{x^{\frac{2}{5}} x^{\frac{3}{5}}} = \frac{5}{x^{\frac{3}{5}}} \quad x^{\frac{3}{5}} = 27$$

$$x^{\frac{2}{5}} x^{\frac{3}{5}} = x^{\frac{5}{5}} = x \text{ or } x = 243. \text{ Ans.}$$

$$\frac{3025}{25} \sqrt{55} \div 5 = 2 = 27 + \frac{1}{2}$$

$$\frac{105}{5} \sqrt{525}$$

$$\left\{ \begin{array}{l} x^{\frac{3}{5}} = 27 \text{ and} \\ x^{\frac{2}{5}} x^{\frac{3}{5}} = x^{\frac{5}{5}} = 729 = 27 \times 27 \\ \hline 756 \end{array} \right.$$

Ex. 19. Reduce $2(1+x-x^2) - \sqrt{1+x-x^2} = -\frac{1}{9}$

Squaring $1+x-x^2$
& subtracting it

Mult^y by 2

Subtracting

$$\begin{array}{r} 2 + 2x - 2x^2 \\ + 1 + x - x^2 \\ \hline 1 + x - x^2 = -\frac{1}{9} \end{array}$$

See again next page.

Change signs & trans^{fer}

$$x^2 - x = -\frac{1}{9} + 1$$

Comp. Square

$$x^2 - x + \frac{1}{4} = \sqrt{1 + \frac{1}{4} - \frac{1}{9}} = \frac{5}{4} - \frac{1}{9}$$

Ex^{ty} root

$$x = \frac{1}{2} \pm \frac{1}{6} \sqrt{41}$$

$$\frac{45 - 4 - \sqrt{41}}{36} = \frac{1}{6} \sqrt{41}$$

It is not perceived why $-\frac{1}{9}$ should not be changed to $+\frac{1}{9}$; no how the above values of x can be found without subtracting $-\frac{1}{9}$ from $1 + \frac{1}{4}$.

Ex. 23. Reduce $\sqrt{2x+1} + 2\sqrt{x} = \frac{21}{\sqrt{2x+1}}$ Clear fractions

$$2x+1 + 2\sqrt{x^2+2x} = 21. \text{ trans^{fer} } 2x+1$$

$$2\sqrt{x^2+2x} = 20 - 2x$$

Introducing the Coeff^{ts}
 \sqrt{x} ~~is~~ under the rad. sign

$$\sqrt{8x^2+4x} = 20 - 2x \text{ Squar both sides}$$

$$400 - 40x + 4x^2$$

$$8x^2 + 4x = 400 - 80x + 4x^2 \text{ Trans^{fer} Quantity}$$

$$4x^2 + 84x = 400 \text{ Dividing by 4}$$

$$x^2 + 21x = 100 \text{ Comp^{ly} Square}$$

$$x^2 + 21x + 110.25 = 100 + 110.25$$

$$x = \frac{-10.5 \pm \sqrt{210.25}}{2} = \frac{-10.5 \pm 14.5}{2} = +4.$$

$$\frac{-10.5}{2} \text{ or } -2.5$$

$$\frac{24}{4} \sqrt{110} = 28.5 \sqrt{1425}$$

* Twice the square root of any quantity, is equal to the square root of 4 times such quantity.

Ex. 24. Reduce $2\sqrt{x-a} + 3\sqrt{x} = \frac{7a+5x}{\sqrt{x-a}}$ *clear fractions*

Transf. $2x-2a$ -
Dividing by 3 - and
Squaring both sides

$$\begin{aligned} 2\sqrt{x^2-a^2} + 3\sqrt{2x^2-2ax} &= 7a+5x \\ &+2a-2x \\ 2\sqrt{9a^2-3x} &= 7a+5x \\ 3a-x &= 7a+5x \\ 3a-x &= 7a+5x \\ 9a^2-3ax &= 7a^2+6ax+x^2 \\ 2x^2-2ax &= 9a^2-6ax+x^2 \\ -x^2-6ax &= 9a^2-6ax+x^2 \\ x^2-8ax &= 9a^2 \text{ Comp. square} \\ x^2-8ax+16a^2 &= 9a^2+16a^2 = 25a^2 \\ x &= 4a \pm \sqrt{25a^2} = 4a \pm 5a \\ &= 9a \text{ or } -a. \end{aligned}$$

Ex 25. Reduce $x+16-\sqrt{x+16} = 10-4\sqrt{x+16}$ *transf. -4\sqrt{x+16}*

Ex 13. Reduce $\frac{x^2}{128x^2-64x^2} = -\frac{1}{32}$

$$\begin{aligned} 128x^2-64x^2 &= -8 \\ 512 & \\ 65536x^2-4a+4096 &= -4096 \text{ Comp. square} \\ (256x^2=64 & \text{ E. root} \\ x^2=64 \div 256 &= \frac{1}{4} \\ x &= \sqrt{\frac{1}{4}}. \end{aligned}$$

$$\begin{aligned} x+16-3\sqrt{x+16} &= 10 \text{ putting -3 under rad. sign} \\ \sqrt{9x+144} &= -x-6 \text{ transf. } x+16 \\ -x-6 & \text{ Squaring both sides} \\ x^2+6x &= x^2+12x+36 \\ 9x+144 &= x^2+12x+36 \text{ transf. } x^2+12x \\ x^2+3x &= 108 \text{ Comp. square} \\ 4x^2+12x+9 &= 432+9 \\ 2x &= -3 \pm \sqrt{441} \mid 21-3 = 18 \div 2 = 9 \\ 41 & \mid 41 \\ & \text{ or } -12 \end{aligned}$$

Ex. 19 again Reduce $2(1+x-x^2)-\sqrt{1+x-x^2} = -\frac{1}{3}$ *Complete square by mult. by 8*

$$\begin{aligned} 16(1+x-x^2-8\sqrt{1+x-x^2}+1) &= \frac{8}{3}+1+\frac{1}{3} \\ 4\sqrt{1+x-x^2}-1 &= \frac{1}{3} \text{ transf. -1 + divided by 4} \\ \sqrt{1+x-x^2} &= \frac{1}{3} = 1+\frac{1}{3} \div 4 = \text{squaring both sides} \\ 1+x-x^2 &= \frac{1}{9} \text{ transf. } x \text{ changing signs} \\ x^2-x &= 1-\frac{1}{9} = \frac{8}{9} \\ x^2-x+\frac{1}{4} &= \frac{8}{9}+\frac{1}{4} = \frac{41}{36} \text{ Comp. square} \\ x &= \frac{1}{2} \pm \sqrt{\frac{41}{36}} = \frac{1}{2} \pm \frac{\sqrt{41}}{6} \\ x &= \frac{1}{2} + \frac{\sqrt{41}}{6} \end{aligned}$$

27 Ex. Reduce $\frac{4x-5}{x} - \frac{3x-7}{3x+7} = \frac{9x+23}{19x}$ 69

$$\frac{156x^3 + 169x^2 - 455x - 39x^3 + 91x}{-66x^3 + 91x^2 - 161x - 27x^3} = \frac{27x^3 + 132x^2 + 161x}{156x^3 + 169x^2 - 455x}$$

$$\frac{90x^3 + 128x^2 - 616x}{-66x^3 + 91x^2 - 161x - 27x^3}$$

Divide by x) $90x^2 + 128x - 616$

2) $90x^2 + 128x = 66$

$48x + 64x = 308$
180

Complete square

$8100x^2 + 11520x + 4096 = 55440 + 4096 = 59536$

$90x = -64 \pm \sqrt{59536}$

$44 \sqrt{1936}$
 $484 \sqrt{1936}$

$180 \div 90 = 2$ the value of x .

Ex. 28. Reduce $\frac{3}{6x-x^2} + \frac{6}{x^2+2x} = \frac{11}{5x}$

$15x^3 + 30x^2 - 30x + 180x^2 = -11x^4 + 44x^3 + 132x^2$

$11x^4 + 59x = -78$

$484x^2 + 3481 = -3432 + 3481$

$22x = 59 + 7 = 66$

$x = 66 \div 22 = 3$

$6x - x^2$
 $30x^2 - 5x^3$
 $180x^2 - 30x^3$

$x^2 + 2x$
 $5x^3 + 10x^2$
 $15x^2 + 30x$

$6x - x^2$
 $x^2 + 2x$
 $6x^2 + 12x$
 $-2x^3$

$4x^3 - x^4 + 12x^2$

$44x^3 - 11x^4 + 132x^2$

Ex 29. Reduce $\frac{(x-5)^2}{4} - 3(x-5) = 40$ Completing the square

$4(x-5)^2 - 12(x-5) + 9 = 160 + 9$ Extra root

$2\sqrt{x-5} = 3 \pm \sqrt{169} = 13$

$2\sqrt{x-5} = 16$

$\sqrt{x-5} = 16 \div 2 = 8$ Squaring both sides

$x-5 = 64$

$x = \sqrt{64} + 5 = 4 + 5 = 9$

Ex. 30. Reduce $x + \sqrt{x+6} = 2 + 3\sqrt{x+6}$ Transposing $2\sqrt{x+6}$

$x-2 = 2\sqrt{x+6}$ square both sides

$x^2 - 4x + 4 = 4x + 24$ Transpose $4x$ & $4x$

$x^2 - 8x = 20$ Complete square

$x^2 - 8x + 16 = 20 + 16 = 36$

$x = 4 \pm \sqrt{36} = 6 = 10$

70. Prob 10-150

A Gent. bought a number of pieces of Cloth for 675 Dollars, which he sold again at 48 Dollars a piece, & gained by the bargain as much as one piece cost him.

What was the number of pieces?

Let x = the number of pieces.
 Then $\frac{675}{x}$ = the price per piece at which the pieces were bought -
 And $48x$ = what all pieces were sold for - By the conditions of the problem - all the pieces were sold for as much more than they cost, as one piece cost. Hence

$$48x - 675 = \frac{675}{x}$$

$$48x^2 - 675x = 675 \quad \text{Completing the square}$$

| | | |
|--|---|--|
| $\begin{array}{r} 432 \\ 48 \end{array}$ | $\begin{array}{r} 1350 \\ 6075 \\ 675 \end{array}$ | $\begin{array}{r} 675 \\ 3375 \\ 4725 \\ 4050 \end{array}$ |
| $\begin{array}{r} 921 \\ 81 \end{array}$ | $\begin{array}{r} 129600x^2 = 129600x^2 - 455625 \\ 455625 \end{array}$ | $\begin{array}{r} 455625 \end{array}$ |
| $\begin{array}{r} 186 \\ 6 \end{array}$ | $\begin{array}{r} 585225 \\ 49 \end{array}$ | |
| | $\begin{array}{r} 146 \\ 1525 \end{array}$ | |

$$96x = 675 + 765$$

$$x = \frac{1440}{96} = 15 \quad \text{The number of pieces. Ans.}$$

Prob 15.

Several Gent. ran up a bill to 175 Dollars, when 2 of them having absconded - the bill was paid by the others: Each one contributing two dollars more, than would have been his share, had the bill been paid by the whole Comp. What was the number of the Comp at first? Let x = the number

$$\begin{aligned} 175 + 10 &= 185 \\ x-2 & \\ 175x - 350 + 10x^2 - 20x &= 185x \quad \text{Expenses} \\ x^2 - 2x &= 350 \quad \text{transfer} \\ x^2 - 2x + 1 &= 351 \\ x - 1 \pm \sqrt{352} &= 6 \pm 18.76 = 24.76 \quad \text{The whole Comp. at first} \end{aligned}$$

Prob. 24. A Gent. bought a certain number of $\frac{3}{4}$ Ounces for 80 Guineas. If he had bought 4 more for the same money, he would have paid one Guinea less for each. What number did he buy? Let x = the number

Then $\frac{80}{x}$ = the price of each: But $\frac{80}{x+4} + 1 = \frac{80}{x}$

Exp^y $80x -$

$$80x + x^2 + 4x = 80x + 320$$

$$x^2 + 4x = 320$$

$$x^2 + 4x + 4 = 324$$

$$x = 2 \pm \sqrt{324} \quad 18 - 2 = 16. \text{ Ans.}$$

$$\begin{array}{r} 25 \overline{) 224} \\ 224 \\ \hline \end{array}$$

Prob. 18. A Gent. bought two pieces of Cloth the finer of which cost $4\frac{1}{2}$ a yd more than the other. The finer piece cost 18£; but the coarser one which was 2 yards longer than the finer cost only 16£. How many yds were there in each piece, What was the price of a yd of each?

Let x = No yds of the finer - then $x+2$ = No yds of the coarser

And $\frac{18}{x} + \frac{16}{x+2}$ will be equal the price of a yd of each - If each of these fractions be severally multiplied by their respective denominators, their products will be equal to the cost of both pieces.

But $\frac{18x}{x}$ makes x equal to 18; whereas $\frac{16x+32}{x+2}$ makes x equal to 16.

$$\text{Then } \frac{18x}{x} + \frac{16x+32}{x+2} + 2 = 2x. \quad \text{Cleaning fractions}$$

$$18x^2 + 36x + 16x^2 + 32x + 2x^2 + 4x = 2x^3 + 4x$$

$$2x^2 - 32x = 72 \quad \text{Dividing by 2}$$

$$x^2 - 16x = 36 \quad \text{Dividing by 2}$$

$$x^2 - 16x + 64 = 36 + 64 = 100 \quad \text{completing square}$$

$$x = 8 \pm \sqrt{100} = 10 + 8 = 18 \text{ No yds of finer - the} \\ 20 - 2 \text{ Coarser}$$

72 Principles of proportion applied to the solution of problems

Ex. 18 There are two numbers whose product is 135
And the diff. of their squares, is to the square of their diff.
as 4:1. What are the numbers? -

Let x & y be the numbers -

$$\begin{array}{ll}
 1 & xy = 135 \\
 2 & - \\
 3 \text{ Expanding } & x^2 - y^2 : x^2 - 2xy + y^2 :: 4 : 1 \\
 \text{Subtract } 4 \text{ Consequently } & 2xy - 2y^2 : x^2 - y^2 :: 3 : 1 \\
 5 \text{ Dividing by } x - y & 2y : x - y :: 3 : 1 \\
 6 \text{ Mult \& Exp \& mean } & 2y = 3x - 3y \\
 7 \text{ Transf } & 5y = 3x \\
 8 \text{ Give } xy = 135 = & x = \frac{5y}{3} \\
 9 \text{ Make 2 last equal } & x = \frac{135}{y} \\
 10 \text{ Subst } 9 \text{ in the last. } & \frac{135}{y} = \frac{5y}{3} = 5y = 405 \\
 & y^2 = 405 \div 5 = 81 \\
 & y = \sqrt{81} = 9. Y. \\
 & x = \frac{135}{9} = 15 X
 \end{array}$$

Prob. 13. In a mixture of rum & brandy, the diff. between the quantities, is to the quantity of brandy, as 100 is to the number of Gall of rum; & the same diff. is to the quantity of rum, as 4 to the number of Gall of brandy.

How many Gallons are there of each?

Let x & y represent the quantities. Then by the Conditions

$$\begin{array}{ll}
 1^{\text{st}} & x - y : y :: 100 : x \text{ by invert \& mean } x - y : 100 :: y : x \\
 2^{\text{d}} & x - y : x :: 4 : y \text{ by invert \& extremes } x - y : 4 :: y : x \\
 3^{\text{rd}} & x - y : 100 :: y : x \\
 4^{\text{th}} & 4 : x - y :: y : x \\
 5^{\text{th}} & x - y : 100 :: 4 : x - y \text{ making 3 \& 4 equal} \\
 6^{\text{th}} & x^2 - y^2 = 400 \text{ mult \& Exp \& mean} \\
 7^{\text{th}} & x - y = 20 \text{ Exp \& root} \\
 8^{\text{th}} & x = 20 + y \\
 9^{\text{th}} & \text{Subst } 20 \& 20 + y \text{ in the } 4^{\text{th}} \text{ proportion } 4 : 20 :: y : 20 + y \\
 10^{\text{th}} & 80 + 4y = 20y \\
 11^{\text{th}} & 80 = 16y \& y = 80 \div 16 = 5. \text{ Subst } 5 \text{ in line of } y \text{ in the } 8^{\text{th}} \text{ equation } x = 20 + 5 \\
 & y = 5 = \text{the Brandy } \& x = 25 \text{ rum}
 \end{array}$$

Prob. 16 There are two numbers, which are to 73 each other, in the duplicate ratio of 4 to 3; and 24 is a mean proportional between them. What are the numbers?

Let x & y represent them
 Then $x:4::4^2:3^2::16:9$ Multiplying Extremes & means
 and $x:24::24:y$
 $9x = 16y$ Subst^y the value of x in the 2^d proportion
 $x = \frac{16y}{9}$

$$\frac{16y}{9}:24::24:y$$

$$\frac{16y^2}{9} = 576 \text{ clearing fractions}$$

$$16y^2 = 5184$$

$$y^2 = 5184 \div 16 = 324$$

$$y = \sqrt{324} = 18 \text{ and of course } x = 576 \div 18 = 32.$$

There are 4 numbers in Geometrical progression, the second of which is less than the 4th by ~~24~~ 24 and the sum of the extremes is to the sum of the means as 7 to 3. What are the numbers?

Let x = the 1st term & y = the ratio — Then, by art. 436

Divid^y by x $x:xy::xy^2:xy^3$
 2 $1:y::y^2:y^3$ — substituting $y+24$ agreeably to the condition of the question

3 $1:y::y^2:y+24$

4 — $25+y::y+y::7:3$ Sum of Ext^s to sum of means as 7:3

5 — $25-y^2::y+y::4:3$ Subtract consequents, from antecedents

6 — $25-2y^2y::y+y::1:3$ Subtracting again

7 — $75-6y^2=3y=y^2+y$ Multiplying Extremes & Means

8 — $7y^2+4y=75$ Transposing & uniting terms

$$\frac{28}{800} \text{ Comp^d Square}$$

$$196y^2+112y+16 = 2100+16$$

$$14y = -4 \pm \sqrt{2116} = 46-4 = 42$$

And $y = 42 \div 14 = 3$. Substituting 3 in place of y in the 3^d proportion

$$1:3::3^2:3^3::9:27$$

1.3.9.27 The numbers sought.

Last problem —
 Geomet^l progression.

74 Prob. 19 - pag 152.

A Merchant bought 54 Gallons of Madeira wine, & a quantity of Lineriff. For the Madeira he paid "x as many shillings by the Gallon, as there were Gallons of the Lineriff: & for the Lineriff, he paid 4/ a Gallon less. He sold the mixture for 10/ a Gallon, & lost 28 pounds 16 shillings by his bargain. Required the price of the Madeira, & the number of Gallons of the Lineriff.

Let x = the number of Gall of Lineriff - then $\frac{x}{2}$ will represent the price of a Gall. of Madeira: and this multiplied by 54 will equal the cost of the Madeira = $\frac{54x}{2} = 27x$

And $\frac{x}{2} - 4$, will represent the price of a Gall of Lineriff; which multiplied by x will equal the cost of the Lineriff. $\frac{x}{2} - 4 \times x = \frac{x^2}{2} - 4x$

$$\text{Whole cost} = \frac{x^2}{2} + 23x$$

But the whole quantity was sold at 10/ a Gallon
Whole quantity = $54 + \frac{x}{2} \times 10$ - and the loss 28.16

$$\text{added} = \frac{540 + 10x}{2} = \frac{x^2}{2} + 23x$$

$$1116 + 10x = \frac{x^2}{2} + 23x$$

$$\frac{x^2}{2} + 23x = 10x + 1116$$

$$x^2 + 46x = 20x + 2232$$

$$x^2 + 26x = 2232$$

$$x^2 + 26x + 169 = 2232 + 169$$

$$x = -13 \pm \sqrt{2401} \quad 49 - 13 = 36 \text{ the value of } x. \frac{36}{2} = \text{the}$$

price of a Gall. Madeira
89 $\overline{) 801}$ 4 shillings - of a Gall Lineriff

$$\begin{array}{r} 23 \\ 13 \\ \hline 36 \\ 4369 \end{array}$$

If the square of a certain number be taken from 40, & the square root of this difference be increased by 10, & the sum be multiplied by 2, & the product divided by the number itself, the quotient will be 4. What is the number?

Let x = the number - then

by the condition, $2\sqrt{40-x^2} + 20 = 4x$

$2\sqrt{40-x^2} + 20 = 4x$ Transposing 20 & dividing by 2

$\sqrt{40-x^2} = 2x - 10$ Squaring both sides

$40 - x^2 = 4x^2 - 40x + 100$ Transposing & uniting

$5x^2 - 40x = -60$ Dividing by 5

$x^2 - 8x = -12$ Completing square

$x^2 - 8x + 16 = -12 + 16 = +4$

$x = 4 \pm \sqrt{4} = 2 + 4 = 6.$

22. Two casks of wine were bought for \$58 & each. One of which contained 5 gallons more than the other & the price by the gallon, was \$2 less than $\frac{1}{3}$ of the number of gallons in the smaller cask. How many gallons were there in each cask, & what the price per Gall?

Let $x+5$ = the number Gall in the larger cask. & x = the smaller
then $\frac{x}{3} - 2$ = the price of a Gallon; & this multiplied into $x+5$
& into x produces $\frac{2x^2}{3} + \frac{5x}{3} - 4x - 10$ and this = \$58

The greater cask $\left\{ \begin{array}{l} \frac{2x^2}{3} + \frac{5x}{3} - 4x - 10 = 58 \text{ clearing fractions - transposing terms} \\ 6x^2 - 21x = 612 \end{array} \right.$
 $= 12 + 5 = 17$
 $\frac{12}{3} - 2 = \$2 \text{ each Gall}$
 $\left\{ \begin{array}{l} 6x^2 - 21x = 612 \\ 144x^2 - 504x + 441 = 14688 + 441 \end{array} \right.$ Completing square
 $12x = 21 \pm \sqrt{15129}$
 $x = 144 \div 12 = 12$
 $\frac{12}{3} = 4$
 $123 \overline{) 729}$
 $48 \quad 249$
 $243 \quad 6$
 $60 \quad 9$
 $90 \quad 9$
 $90 \quad 0$

76 Prob. 23. In a parcel which contains, 24 coins, of Silver & Copper; each Silver coin is worth as many Cents as there are Copper coins; And each Copper coin is worth as many cents, as there are Silver coins: And the whole are worth \$2.16—How many are there of each?

If $x =$ Silver Coin
 $24 - x =$ the Copper

$$\left\{ \begin{array}{l} 24 - x \times x \\ \hline 24x - x^2 = \text{the value of sil. coin} \\ 24x - x^2 = \text{also, the value of the Cop. coin} \end{array} \right.$$

$$48x - 2x^2 = 216 \text{ Change signs}$$

$$2) \quad 2x^2 - 48x = -216 \quad \text{transf. d.}$$

$$x^2 - 24x = -108 \quad \text{Complete square}$$

$$x^2 - 24x + 144 = -108 + 144$$

$$x^2 - 24x + 144 = 36$$

$$2x = 24 \pm 12 \quad \text{Extr. root}$$

$$x = \frac{24 \pm 12}{2} = 18 \text{ Sil. } \& \text{ 6 Cop. coin}$$

Prob 24 - A Person bought a certain number of
 open for 80 Guineas. If he had bought 4 more for
 the same money, he would have paid a guinea a
 head less - What was the number?

Let x = the number. By the conditions,

$$\frac{80}{x+4} - \frac{80}{x} = 1 \quad \text{Clearing fractions}$$

$x+4$ x
 $80x = 80x + 320 - x^2 + 4x$ Transferring constant terms
 $-x^2 + 84x + 320 = 80x$ Changing all the signs
 $160x$ for $320 = 320$ when $x^2 - 4x = 320$ Completing square
 $2080 - 80 = 84 -$ $x^2 - 4x + 4 = 320 + 4$
 $x = 2 \pm \sqrt{324}$ 18
 $28 \overline{) 224}$
 $x = 2 \pm 18 = +20$
 $\text{at } 16 \neq$ this is the right value of x

Prob. 25 - page 164

There is a certain number consisting of 2 digits. The left hand digit is equal to 3 times the right hand digit; and if 12 be subtracted from the number itself, the remainder will be equal to the square of the left hand digit. What is the number?

Let x = the left hand digit, & y the right hand digit - note as the local value of figures increases in a ten fold ratio from right to left - the number required = $10x + y$

By the conditions of the problem - $x = 3y - 1$
and $-10x + y - 12 = x^2 - 2$

add the 2 Egs - $-10x + 4y - 12 = x^2 + x - 3$

Multy 2 by 4 - $40x + 4y - 48 = 4x^2 - 4$

Subt 3rd from 4th - $36x - 36 = 3x^2 - x - 8$

Transf. 3rd uniting $3x^2 - 31x = -36$

Complete square $36x^2 - 372x + 961 = 432 + 961$

Ext root $6x = 31 + 23 \div 6 \left\{ \begin{array}{l} \pm \sqrt{529} \end{array} \right. \begin{array}{l} 23 \\ 4 \end{array}$

$x = 9$ the left - $43 \overline{) 129}$

$x + y = 93$ - hand digit - and $y = 3$

Prob. 26 page 165.

If a certain number be divided by the product of its 2 digits, the quotient will be 2. and if 27 be added to the number, the digits will be inverted. What is the number? Let $x + y$ = the digits - Then

by the conditions of the problem $\frac{10x + y}{xy} = 2 = \frac{10y + x}{2xy}$ and

* the value of the fraction = $y - 3$

$\frac{1}{3} \cdot 8y^2 - 153y = -270 = 8^{th} \text{ transf. \& inverted}$
 $6y^2 - 51y = -90 \text{ Conf. eqn}$

$144y^2 - 224y + 201 = -2160 + 2601$

$12y = 51 + 21$

$y = \frac{72}{21} = 6$ Substituting 6 in

in 1st Eq. x is found to be

equal to 3. So the Number sought is 36.

Let 1st from 2 - $27 = 10y + x - 2xy$

Transf. - $2xy - 2xy + 27 = 10y + x$

By the 2nd Eq $x = 9y - 27$ and substituting this

value for x in the 2nd Eq. $2xy + 27 = 11y - 3$

By last Eq $x = \frac{11y - 3 - 27}{2}$

By making 2nd & 6th equal $11y - 3 - 27 = 9y - 27$

2nd $99y - 27 - 243 = 18y^2 - 54y$

Divide 90 into 4 such parts that the 1st + 2, the 2^d - 2, the 3^d x by 2, & the fourth divided by 2, shall all be equal.

Let x, y, z be then of the parts, & $90 - x - y - z$ the fourth.

By the conditions, $x + 2 = y - 2$

| | | |
|------------------------|---------------------------|---|
| $x = 18 + 2 = 20$ | 2 — $x + 2 = 22$ | clearing fractions transf. — 2 multy. 2 Eq by 2. Or, add together last 2 members of 1 st Equation — making sum = 4.7 |
| $y = 22 - 2 = 20$ | 3 — $2x = 90 - x - y - z$ | |
| $z = 10 \times 2 = 20$ | 4 — $5z = 90 - x - y - z$ | |
| $w = 40 \div 2 = 20$ | 5 — $4z =$ | |
| 90 | 6 — $9z = 90$ | |
| | 7 — $z = 10$ | from which all the others are easily found. |

Jan. 1845.

Philadelphia Paper

A Lady being asked her age — ans. that the number expressing it, consisted of two figures or digits — the left hand one of which is 5 less than the right hand one; and that $\frac{4}{9}$ of the number is less than the product of the two digits by twice the left hand digit. What is her age?

Let x equal the left hand & y the right hand digit. Then $x = y - 5$. But the locality of x gives it a tenfold value. Hence the number sought = $10x + y$ and by the conditions $\frac{4}{9}$ of $(10x + y) + 2x = 7xy$.

that is $4(10x + y) + 2x = 7xy$

2^d Expanded $40x + 4y + 2x = 7xy$ multy by $7x$

3^d $7x^2 = 7xy - 35x$ Subt 2^d from 3^d

4th $7x^2 - 54x - 4y = -35x$ mult 1st Eq. by 4

5th $+4y - 20 = +4x$ add 4 & 5

6th $7x^2 - 54x - 20 = -31x$ Transf. & unit

$7x^2 - 23x = 20$ Comp 7 squared

$196x^2 - 644x + 529 = 560 + 529$ Expt 7 root

$14x = 23 +$ 2 1089 33

33 2 189

$x = 56 \div 14 = 4$ 63 189

and $y = 4 + 5 = 9$ so 49

is the number sought.

with \$100 Dollars

79

A Gentleman sent his servant to purchase Cows at \$10 a head. Sheep at \$1 each a head & fowls at one Shilling a head so that the number of Animals bought should be equal to the number of Dollars paid for them. What were the numbers of each sort?

It is obvious that the sheep, costing a dollar a head, may be left out of the investigation: And then the question resolves itself into this— "What 2 numbers are those, whose sum is equal to the product of one of them multiplied by 10, added to the quotient of the other, divided by 8?" Let $x = \text{no. cows}$ & $y = \text{no. fowls}$

Then $x + y = 10x + \frac{y}{8}$ according to the conditions of the question.

8 Clairy pations

$$8x + 8y = 80x + y \text{ transp. } y \text{ unitary}$$

$7y = 72x$ - Here the coefficients designate the numbers sought

$$\frac{72x}{77} = \frac{7y}{77}$$

That is

7 Cows at \$10 = \$70 and

72 fowls at 1/8 = 9

79

21 Sheep at 1/2 = 10.5

So if the Cows had been bought at \$15 a head

& the fowls at 2/8 on the same conditions

Then $x + y = 15x + \frac{y}{4}$ where the no. Cows is 3 x 15 = 45

and of fowls $56 \div 4 = 14$

$$4x + 4y = 60x + y$$

$$3y = 56x$$

Animals 59 = 59 Dollars

Corollary - If any unknown quantity be multiplied by any number whatsoever, be made or found equal to any other unknown quantity, also multiplied by any other number whatsoever - Then, universally, the value of the latter quantity will be equal to the multiplier or coefficient of the former quantity; and the value of the former quantity will be equal to the multiplier or coefficient of the latter: Or, the quantities will be equal multiples of those coefficients. Thus if $x = 5y$ - then $x = 5 \times y = 1$ or $9y = 11x$ - $x = 11 \div y = 9$ or

in the 1st Ex. if y had equaled 11, then x would have = 55

It is plain that if $x = 2, 5, 10$ or $1000y$; then x will be equal to 2, 5, 10 or a 1000, $xy = 1$

From the Conditions of a problem, it can readily be determined whether the Coefficients be the true numbers sought, or whether the true numbers be some equal multiples of them - Thus in the 7th equation Ex. 10 - page 72, Where $3x = 5y$ & where the Conditions require that the product of

$$xy = 135 \text{ &c. Multiplied by } 3$$

$$x = 225 - 81 : 36 : 4 : 1$$

$$144 : 36 : 4 : 1$$

March 21. 1844.

Casuistry

Allow, that conscience rightly regulated,
Implies, conformity to rules, well stated:

That every duty, scrupulously to fulfil,
Claims strict obedience, to the enlight'nd will;

Then, must it not unquestionably follow,
That what's hereto opposed, must clearly needs be hollow?

Suppose obedience challeng'd to a strict command,
That we should neither sit, lie, walk, ~~nor~~ stand;

Or, the performance, of some mental function
Should be enjoined, where clearly the injunction
Invades the undoubted province of the will;

Can it be urg'd, duty claims obedience still?

Then to work hard, & not grow faint nor weary,
To endure most wanton wrongs, & yet feel cheery,

To love what's hateful, or to eschew what's good,
Are, or may become duties, not to be withstood;

Provided, an ordinance, be therefor assign'd:

Mooted divine or papal, to soothe the pleasant mind.

Commands, like these, so abhorrent to our nature!

Can they aught else be, than very small potatoes?

Suppose again - our firm belief's enjoined
Of something, not prov'd, nor can be, nor defin'd:

Now, what's called God, in times, now grown remote,
Familiar was with man, & to him spoke & wrote:

Once in particular, ~~laid~~ ^{back} his backside to Mose,
Hid in a cleft, as by it pass'd, he goes.

And oft'ns every now & then, would stray,
To visit man, eat kids, & spend an holiday.

Not, that all men alike, his visits shared;

On some he smiled; on others fiercely glared.

Just merely by this means or that, to indicate
That God, like man can love as well as hate.

But rather, ~~since~~ cherishing a long ponder'd notion
That his true glory turn'd on man's devotion:

And that, could he but find a model man or nation
To would endure that glory thro' his wide creation.

Thence far & near look'd round to speed in Carran
A man called Abram ^{him} whose wife was barren.

This man seem'd well to do, yet sorely felt the need
Of things ~~which~~ ^{which} ~~he~~ ^{he} ~~was~~ ^{was} ~~in~~ ⁱⁿ ~~need~~ ^{need} of his own & Sarah's seed.
Their own endearments failing, to accomplish their intent,
Abimelech & Pharaoh they resolv'd to circumvent -
Already had they experiment'd upon the dusky Neger
But they did not like the lud, because he was half Neger.

So they pondered well the matter, till they clearly could divine,
That not a royal heir, they'd get ~~handed~~ of Sheep & Mine.

Our allowance being made for the times as they were, then
General honesty prevailing, but few left handed men.

Thus easily Abram found it, sister for wife committed

To come it over there, kings, at the expense of being committed.

V V V

Ten times the square root of a certain mean be
Together with one eighth of itself, & six — Given to find number

$$10\sqrt{x} + \frac{x}{8} + 6 = x \quad \text{Transp. & c}$$

$$10\sqrt{x} = \frac{7x}{8} - 6 \quad \text{Clear fractions}$$

$$80\sqrt{x} = 7x - 48 \quad \text{Sqr. both}$$

$$6400x = 49x^2 - 672x + 2304 \quad \text{Sides}$$

$$49x^2 - 7072x = -2304 \quad \text{Transp. 7th}$$

$$\text{Multi. } 196 = 4 \text{ times Co. Eff.}$$

$$\begin{array}{r} 294 \\ 441 \\ 49 \cdot \end{array} \quad \begin{array}{r} 42432 \\ 63648 \\ 7072 \end{array}$$

$$9604x^2 = 1386112x + 50013184$$

$$\begin{array}{r} 81 \\ 1504 \end{array} \quad \begin{array}{r} 98x = 7072 \pm 7040 \\ + 7040 \end{array}$$

$$x = 14112 \div 98 = 144$$

$$\begin{array}{r} 431 \\ 392 \\ 392 \\ 392 \end{array}$$

Again Let $x^2 =$ equal the number — Then

$$10x + x^2 + 6 = x^2 \quad \text{By transposing & uniting}$$

$$10x = x^2 - 6 \quad \text{Clear fractions & stating terms}$$

$$80x = 7x^2 - 48 \quad \text{or } 7x^2 - 80x = 48 \quad \text{Completing Square}$$

$$196x^2 - 80x + 6400 = 1844 + 6400$$

$$14x = 80 \pm 57744$$

$$x = 80 + 58 \div 14$$

$$16 \overline{) 68} \mid 12 = \text{Ans.}$$

Coefficient of the highest power of the
unknown quantity — & add to both
sides the square of the Co. Eff. of the Lower P.

by Multi. equation by 4 times the

$$\begin{array}{r} 196x^2 - 7072x = -2304 \\ 196x^2 - 7072x + 49504 = 49504 - 2304 \\ 196x^2 - 7072x + 49504 = 49504 - 2304 \\ 196x^2 - 7072x + 49504 = 49504 - 2304 \end{array}$$

$$\begin{array}{r} 49561600 \\ 49 \\ 1404 \end{array} \quad \begin{array}{r} 7040 \\ 5616 \end{array}$$

What is the true & legitimate import of the term, Religion? Does it denote a tie, bond, or obligation, duty, & something that ought & must be done? Is it not applicable to all moral beings, universally? and is not precisely synonymous with duty? It seems probable that its primitive meaning was more limited simple and precise, than its present, and that it was used only in a physical sense as to bind a bundle, or to tie with a cord: and that afterwards when a word was needed to denote the idea, of what men must or ought to, do as moral beings, this word came to be used for this purpose also; and thus like many other words, through the poverty of language acquired a twofold meaning. However this may have been, is perhaps of comparatively little moment now, other than as showing how the original meaning of words may gradually & insensibly become modified & essentially perverted by popular use & acceptance.

The word 'Religio' amongst the ancient Romans, from whose language this word is obviously derived denoted certain rites & ceremonies to be observed & performed in honor of their gods: some of these consisted in assembling together in magnificent temples, & there publicly addressing their prayers, & singing sacred songs to the praise & glory of their gods: whilst other services consisted in taking care of the sacred chickens, watching the flight of birds, inspecting the entrails of slaughtered animals offered in sacrifice, & in interpreting prodigies & omens. All which were done by the Priests, in order to ascertain the will of the gods & thereby secure their favour. The feelings & emotions which gave birth to these rites, & which accompanied the performance of them were uniformly characterized by great gravity veneration & awe.

This was religion amongst the Romans: and amongst other contemporary nations, other observances, also purporting to be religious, were greatly diversified. But the question is, what is religion now? Assuming that the word denotes ~~what~~ the idea of what men must or ought to do; it is plain that the question involves an inquiry concerning the intellectual & moral character & constitution of man: and the method of con-

84] -ducting such an inquiry or investigation must obviously be very much as tho' the inquiry were "what is this mineral plant or animal good for? and what use was it intended, or is it fit, to subserve?" The true answer to a question of this sort, must depend on a right understanding of the properties & qualities, powers & faculties that belong respectively to these several subjects. Just so must the true answer, to the question "what is religion" depend on a right understanding of the moral constitution of man. To say - man is a moral being without having clear & distinct ideas of what is meant by the words 'good & evil' 'right & wrong' is sheer verbiage. Hence it is of the first importance to get clear & well settled ideas of these several terms. We certainly can have no definite conception of a moral being without distinct & determinate ideas of the chief elements that make him such. The magnet needle is affected by iron.

This we know. The motion produced by it, is purely physical & is ascribed to attraction. But what attraction is, or that power which is thus denominated, we know nothing.

In regard to moral beings, & moral actions, is not our knowledge in some degree alike circumscribed? We certainly know only that such beings as we are, are capable of acquiring knowledge of enjoying happiness, & of suffering misery; are endowed with imaginative faculties & sundry passions, affections, & emotions.

These several capacities, faculties & endowments we suppose characterize a moral being (theoretically); and also, practically when duly developed & matured by right training & education. For he, only is a moral being strictly considered, who does ^{only} what he ought.

But this, according to the views now advocated, makes him also a religious being: For being tied firmly, & put under bonds by his native faculties & endowments; he can perform no action at all, but in as far as he is thereby prompted; and if he do not act morally & religiously when he does what he sincerely considers right; it would seem to follow, that religion is involved in inextricable obscurity.

Again motives necessarily precede action; for without them, there could be none. But what are motives? Clearly, whatever causes or induces motion. But since the whole ^{outer} world is a mingled mass of good & evil, & nought beside - with this qualification viz. all good if rightly used; or, all evil, if abused. And since also

Moral & Religious Beings are, by their native endowments, exactly adapted to the external world, with intellectual faculties to discriminate between the good & evil, & with a correspondent faculty of being intensely affected, ^{as they are or abuse} accordingly, it: (the world). It seems difficult, ^{if rationally to consider} that there can be any other motives to voluntary human actions, than such as arise from considerations, ~~of the good & evil~~, of good & evil either present or remote.

If the attempt to teach one, blind from his birth, the art of ~~Painting~~ or another, deaf, that of music, be rightly deemed preposterous, because of the want of those faculties on which success in these arts, essentially depends: Can it be any less so, to attempt to teach matters & things appertaining to a spirit world, to such as are wanting in all those faculties & means, necessary to enable to test the truth, & appreciate the worth, of what may be thus taught?

If human Beings are incapable of learning those arts, in which the instincts of some of the lower animals enable them to excel; is it not the height of absurdity to attempt to indoctrinate them, in those arts, pursuits, scenes of enjoyment, & suffering, which are supposed to be apposite to an order of spiritual existences in a world, unknown?

Now let us consider, what such Beings as we are, can, must, & will be sought to do; for, since religion, whatsoever may be its true import, must necessarily be included within this category; we shall not fail to detect that import, provided, we can clearly & satisfactorily determine what sorts of actions necessarily fall within the scope of the human faculties.

I suppose it will be conceded by all, that man is endowed with the faculty of discriminating between right & wrong; and as this conception is independent & regardless of creeds & localities - it must be held universally true of all men of sane minds: it will be further conceded, that all men constitutionally & by nature are characteristically alike; & yet marked by individual peculiarities. Now notwithstanding this unanimity of sentiment, in regard to this discriminating faculty: there is a no less marked contrariety amongst what are considered cultivated minds, in regard to what constitutes right & wrong, & what those elements are, betwixt which the moral faculty discriminates: and here is the grand difficulty to be wrestled with; nay, the only one - Religion or morality, out of the question - all other human actions sink into comparative insignificance.

Beyond all question, if man were bereft of the capacity for Happiness - it is inconceivable, that, tho' the whole Universe should resound with songs of joy, or the howlings of despair of other beings, they would fall alike on his ear, with the same passionless indifference.

Hence it is evident that human Happiness is the "primum mobile", the first & deepest principle in the human constitution, the sole end of all human effort.

Man, then, can distinguish good from evil: could he make this distinction, not so much voluntarily, & because he chooses to do so, but rather necessarily, & because he cannot do otherwise. It does not rest with him, nor at ^{all} depend on his Will, that the various objects of sense, minister to his happiness, or, on the contrary, to his misery. The true reason or cause lies further back, & admits of no other explanation, than that such is the primitive constitution of things, the established order of nature. Man is provided with no other

means for finding out these natural relations, nor of their availability to Happiness, than that of knowledge, derived from observation & experience.

And just so far & so fast as his knowledge advances, to the same extent does he devote himself to the pursuit of what he esteems good, and to the avoidance of evil: Now, thus far man acts as he MUST: Can he avoid preferring good ^{Swiss} murens & cabbage as articles of diet, to skunk cabbage & wild turnips - a bowl of strawberries & cream, to one of curdy & rotten eggs? It is by precisely by the same means - that is - by experience that we arrive at the knowledge of right & wrong in human actions; & precisely for the same reason that we approve the one, & condemn the other, that is - because of their respective tendencies to subserve, or subvert human happiness. Man in the outset, finds himself in a state of utter destitution, both of ability to do anything, & of all knowledge of the ^{means} requisite for his subsistence & enjoyment. He has been unconsciously awakened into life, & introduced into a world, full of incitements to his activity, & in such order as his active faculties shall be ~~come~~ developed - Thus far, he has been exclusively in the hands, as it were, of the Potter, & wholly subject to the guidance of invisible principles within, combined with the influence of circumstances exterior to himself; and until it can be shown how & when he becomes emancipated

from the government of these principles & influences; I see not why it should not be admitted, that it is continued with unabated energy thro' the whole period of life.

It is not intended, by any means, to either deny or ^{over}throw the responsibility, of which all are conscious; but simply to enquire after, & to ascertain its nature & extent (taking it for granted, that religion & accountability are commensurate & coextensive) And keeping in view also, that we are now speaking of such minds only as are supposed to be ordinarily enlightened. Now what are the facts & conceded truths in every case of a wounded conscience? Is it not clear that some act has been done, or resolved on, which, at the moment, viewed under false lights, promised to be beneficent; but which, when subsequently reviewed under better lights, the understanding disapproves? And hence the mortal anguish, termed Compunctious visitings of a violated conscience? And is not Repentance the natural consequence of sober second thoughts, & a dispassionate review & grave reflection — the very fact of an uneasy conscience, which constitutes the chief proof of the existence of a religious faculty, necessarily involves the idea, that conscience is itself the rule & measure of religious obligation; and that whilst the conscience is at ease, the demands of religion are satisfied. But conscience is only the felt approval or disapproval of the decision of the understanding; and since, as before shown, nothing but the consideration of good & evil, either present or remote can possibly affect or in any way modify such decision; it seems to follow that the whole of responsibility is resolvable into the knowledge of good & evil as the indispensable means of attaining the chief end of human existence.

If man be as commonly supposed, wholly & absolutely dependent: How & for what can he be considered justly responsible at all, especially to that Power on which he is dependent? Does not the idea of responsibility involve that of absolute freedom? the ideas of absolute dependence & accountability who can reconcile? Is it possible for us to conceive of a Potter, who should dash the head of a galloning, that he had made, that it should stretch itself to the capacity of a barrel — and that he should dash it to pieces for its disobedience? Is it at all more conceivable, that a sensitive & intellectual piece of Machinery can enlarge, diminish, or in any other way modify its powers & capacities, than can that of a jug? It is obvious, that the idea here involved is none other, than that of self creation: For to suppose that

a created being exercising functions, other than such as properly belonged to, & were within the scope of its primitive endowment, is to suppose him acting thus far in spite of constitutional restrictions, independently of his "OWN WORK". But to avoid further entanglement in the meshes of liberty & necessity, let us return to the question of responsibility, regarded in such light as renders it at all intelligible: and here is it not obvious that the power or principle, to which we are responsible, is within, if not a part of ourselves? Is not the whole process of wrongdoing, of reflecting on it, of penitence therefor, of carefully guarding against a repetition of the evil act, & of mental quiet & security that we succeed, wholly & exclusively limited to ourselves? We are certainly not cognizant that any being exterior to ourselves, has any thing to do with any part of this process: and inasmuch as this process is to be met with amongst all sorts of men, irrespective of their superstitious creeds, & as well amongst those who worship the Devil as those who worship God; we seem compelled to the conclusion, that the whole theory of accountability is resolvable into the subtle & mysterious operations of our own minds. The idea here suggested must not be understood either to exclude, or to detract from the doctrine of the Divine Agency in the absolute government of the Universe.

The constitution of the world as we find it, & the general Providence, or Order of Nature by which it is sustained are considered as affording ~~an~~ ^{an} ~~affording~~ ^{affording} us all the light we possess, & all that is needful to enable us to live in such a manner, as to ensure the greatest glory to God, & the greatest amount of happiness to man: This last however is, that with which we are at present concerned - the only legitimate end of all scientific research. The following unquestioned facts, ^{the} considerations inseparable from them, seem to me to have a direct bearing upon the proposition here stated; and to be conclusive as to its truth. The first of these facts, is the actual & real existence of a Code of Laws (supposed for the most part, to have God for their Author; & by so much the better if the supposition be true) called "the Laws of Nature", which are constant & immutable in their operation in their operation & universal in their scope & application; whose irresistible energy and authority are embodied in principles, which so pervade & vivify all forms & conditions of being, that no event, great or small, does or can take

place, but by thro' their agencies. Now the first prominent consideration, inseparable from this code, is, that the whole circle of modern science is exclusively based upon its real existence & supreme energy; and so much so, that no event NO transpires, worthy the attention of thinking minds, which is not spontaneously regarded as an inevitable sequence in the Order of nature: And such is the degree of confidence among such minds, that whether the event can be traced to, or clearly connected with its supposed antecedent or not, it matters not, it is still regarded as a natural event; and that the principle that gave birth to it, as having thus far, eluded observation. Such amongst the yet unexplained occurrences, may be reckoned the "Aurora Borealis", Aerolites, Camphor, dropped into a basin of water, & its immediate state of quiescence by means of a single drop of any aromatic fluid &c &c

There is another consideration, that like Arons rod seems to swallow all the rest. It is that of the moral & intellectual constitution of man, viewed in connexion with the constitution of things. Here we find ourselves introduced into a world, teeming with good & evil, promiscuously intermingled; & subject to no prohibition as to Appropriation & enjoyment, save that of knowledge, previously to be elaborated by dint of observation & experience. We find herein precisely such incitements to activity, as are requisite to awaken & call forth our dormant mental powers, without which we should remain at best, but doubtful competitors for precedence with the OURANG-OUTANG. Seeing that such beings as we are, can act at all, only by the impulse of motives, we are here supplied with a key to their arcana, the whole machinery of human actions is hereby revealed. The Desire of Happiness incorporated, or provided for in the constitution of man; and the KNOWLEDGE of GOOD & EVIL, the indispensable MEANS of gratifying that desire. These much may be regarded as clearly indicated by the established Order of Nature.

A brief recapitulation of the foregoing (let it be confessed disultory & ill arranged) facts, it is hoped, will render more perspicuous our idea in what true religion consists — and in the first place, the idea of the tie or bond, supposed to be implied by the force of the term, is to be found ONLY in the stringent and unyielding force of the laws of Nature. And in the next place, these laws as arbitrarily impel us to seek after good as a means of

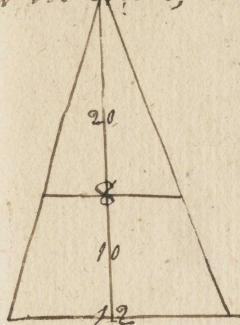
20th Happiness, as they do, inert matter, to obey the force of gravitation; or a lower animal or vegetable, its instincts.

We enjoy or possess perfect physical liberty, within the limits prescribed to our physical powers, to do whatsoever we will; & yet we are absolutely constrained to will whatsoever shall appear to our understandings, at the moment of willing (all things considered, particularly present & remote enjoyment) as the preferable good.

The order of Nature or Providence affords abundant means for promoting human happiness; the chief of which is knowledge. Without knowledge, all other means are as likely to be converted into curses as blessings. The incentives to knowledge, are desires & wants.

True Religion enjoins obedience to the laws & institutions of nature; and implies such provision of their operation, as to enable us in some degree to modify & controul ^{of their} operation.

A Truncum of a Cone is ten feet in length, & its bases 12 feet ¹²³
 & 8 feet in diameter. Where shall it be cut off by a section parallel
 to its ends, so that the parts shall be equal to each other?



$$12^2 \times .7854 \times 10 = 1130.9760 - 335.1040 = 795.8720 = 397.9360$$

$$8^2 \times .7854 \times 6\frac{2}{3} = 335.1040 + 397.9360 = 733.0400$$

$$335.1040 : 733.0400 :: 20^3 : 17500$$

The cube root of which is 25.962744
 25.962744
 from 30
 leaves 4.03725534 multiplied by
 12
 48.4470636 subtracted from
 120.

Answer in inches 71.5529364

Two men buy 80 lbs of Beef at 4 cents a pound \$3.20 cents
 On dividing it between them, one takes 50 lbs, & the other
 30 lbs: But it is considered that the larger quantity is
 worth $\frac{1}{2}$ cent a pound more than the smaller. How much
 shall each pay?

Suppose the better part 2 Sup.

$$\begin{array}{r} \text{worth } 4\frac{1}{8} \text{ cents} \times 50 \text{ lbs} = 212\frac{1}{2} \\ 3\frac{3}{4} \times 30 = 112\frac{1}{2} \\ \hline 3.25 \\ 3.20 \end{array}$$

1st Sup 4 $\frac{1}{8}$

$$\begin{array}{r} 4 \\ 21\frac{1}{4} - 2 \\ \hline 20\frac{5}{8} \end{array}$$

2 Sup. 4 $\frac{1}{8}$

$$\begin{array}{r} 4 \\ 20\frac{5}{8} \\ \hline 20\frac{5}{8} \end{array}$$

2 Error - 5

$$4\frac{3}{16} = \frac{67}{16} \times 50 = \frac{3350}{16}$$

$$3\frac{11}{16} = \frac{59}{16} \times 30 = \frac{1770}{16}$$

Sum 5120

16 5120

3.20 Proof

Sum of 10 24 1 $\frac{7}{8}$ sum of the products

Prices { 4 $\frac{3}{16}$ 15 \div 10 = 15 or 3 $\frac{3}{16}$
 3 $\frac{11}{16}$ 8

What is the true & legitimate import of the term "religion"? Does it denote a tie, bond, obligation or duty—something that ought, that must be done? Is it not applicable ^{universally} to all moral beings; and is it not synonymous with duty? It seems probable that its primitive meaning was more limited, simple & precise, than its present; and that it was employed only in a physical sense—*es.* to bind a bundle, or to tie with a chord &c. And that afterwards, when a word was needed to denote the idea of what men must or ought to do, this word came to be used for this purpose also, ^{thus} like many other words thro' the poverty of language, acquired a double meaning. However this may have been, perhaps is of comparatively little moment now, otherwise than as showing how the original meaning of words may gradually & insensibly become modified & essentially, perverted by popular use & acceptance.

The word "religio" amongst the ancient Romans, from whose language this term is obviously borrowed, denoted certain rites & ceremonies to be observed & performed in honor of their gods: Some of which consisted in assembling together in splendid temples; in addressing ^(their gods) their prayers, & in singing hymns to them; whilst other services consisted in taking care of the sacred chickens, watching the flight of birds, ~~in~~ inspecting the entrails of slaughtered animals offered in sacrifice, & in interpreting ^{of prodigies &} omens; all which were done by the Priests, in order to ascertain the will of the gods & to secure their favour. The feelings & emotions which gave birth to those rites, & which accompanied their performance were characterized by gravity, fear, reverence, veneration & awe.

This was religion amongst the Romans. Amongst other contemporary nations, ~~these~~ rites ceremonies & observances, also purporting to be religious, were greatly diversified.

But the question is, what is, & what constitutes, religion now? Assuming that it is a term, used to denote ^{the idea} of what men must, or ought to, do; it is plain that the question ^{involved} is an inquiry concerning the intellectual & moral character & constitution of man: What he can, must, ought to, & will do, considered as a sensitive moral & intellectual being. And the method

132] of conducting such an investigation must, obviously be very much as tho' the inquiry were - What is this, or that mineral or plant, good for? or what use was it intended; or, is it to subserve? Or what can & will this or that kind of lower animals, do? and what useful purposes can they be made subservient? The true answer to any such questions must depend on a right understanding of the properties & qualities, powers & faculties, that belong respectively to these several subjects. Just so, must the true answer to the question - What is religion? depend on a right understanding, of the moral constitution of man.

To say - man is a moral Being - without having clear & distinct ideas of what is meant by the words "good & evil", "right & wrong", is more verbiage. Hence, it is of the first importance to get clear & well settled ideas of these several terms. We certainly can have no definite conception of a moral Being without distinct & determinate ideas of the chief elements, that make him such. The magnetic needle is affected by iron. This we know. The motion produced is purely physical, & is designated by the word "attraction". But what attraction is, or that power which is thus denominated, we ^{know} nothing. This is the limit of our knowledge; and our reason does not enable us to go beyond it.

In regard to a moral being, is not our knowledge in some degree alike circumscribed? We certainly know only that beings like ourselves are capable of knowledge, & happiness & misery. Beyond this (our capacities for knowledge & happiness) can we, do we know any thing, of ourselves? In what these capacities consist, we are as much in the dark, as we are in regard to gravitation & electricity. In addition to these capacities, we may add - that we are endowed with powers of imagination, with various affections, and sundry emotions. These several capacities, we suppose, characterize a moral Being: Or, rather, are essential to such a being; and really constitute a moral Being, only, when duly educated & developed. So, he only, who does what he ought, can justly be regarded as moral or religious. To draw forth, to nourish, to invigorate and to train these several faculties, to a state of healthful maturity constitute the sole purpose, the only end of education.

If the attempt to teach one, blind from his birth, the art of painting; or another, deaf, that of music, be rightly deemed preposterous, because of the want of those faculties on which success in these arts depends; can it be any less so, to attempt to teach matters & things, appertaining

to a spirit world, to such as are wanting in all those faculties, necessary to enable them to test the truth, & to appreciate the worth of what may be thus taught? If human beings are incapable of learning those arts, in which the instincts of the lower animals enable them to excel; is it not the height of absurdity to attempt to indoctrinate them in those ^{other} arts, pursuits, scenes of enjoyment or suffering, which are supposed to be ^{to be} appropriate to an order of spiritual existence in a world unknown?

Let us then proceed to consider what man, endowed as they are, can, will, & ought to, do. ^{for} since religion, whatsoever may be its true import, must necessarily be included within this category, we shall not fail to detect that import, provided we can clearly & satisfactorily determine what actions necessarily fall within the scope of the human faculties. How much or little soever our moral & religious conduct may be affected by the action of the heart & lungs; & those movements, & continued transformations in the material parts of our bodies, which properly belong to the vital economy; I shall omit for the present, to notice ^{because} they are powers manifestly distinct from, & independent of ourselves & therefore not ~~existing~~ ^{concerning} within the purview of the present inquiry.

It is conceded by all, that man is endowed with the faculty of discriminating between right & wrong: And this conception is wholly independent & regardless of creeds & localities. It may hence be regarded as true of all men universally; that is - of all men of sane minds.

It is also conceded that all men are constitutionally & by nature, alike in their general characteristics; tho' marked by individual peculiarities.

And yet notwithstanding this unanimity of sentiment in regard to the discriminating faculty; there is a no less marked contrariety amongst what are considered cultivated minds, in regard to what constitutes right & wrong, & what ~~are~~ ^{are} those elements, between which the moral faculty discriminates.

Now is there any other conceivable way of adjusting this conflict of opinions, than ^{by} such means as are derivable from the knowledge of Good & Evil?

If not, then it would seem, that good & evil are the primary & fundamental principles which constitute ^{equally & alike} the foundation of right & wrong, of morals, of duty, of religion & of human happiness; for if the capacity for happiness & misery be supposed to be abstracted from man; it is utterly inconceivable, that there could be aught left, to tie or attach him to existence: So that, whether ^{with} the ^{universe} surrounded with songs of joy, or, with the howlings of despair; they would fall upon his ear with the same passionless indifference. } see page 128.

A & B bought 300 Acres of Land for \$6000 each
 paying \$3000 each - On dividing it was found that the part
 which was allotted to A was worth ^{cents} twenty five an acre more
 than that allotted to B. How ought it to be divided?

Let X equal B's part & Y the price per acre - then $XY = 3000$
 3000 dollars - A's part will be 300 acres - $X: Y = 300: X = \frac{300}{Y}$

Substituting the value of X -
 $(300 - 300)X (Y + \frac{3}{4}) = 300$

$$300Y - 300 + \frac{900}{4} - \frac{900}{4Y} = 300 \text{ transp}$$

$$300Y - \frac{900}{4} = 300 + \frac{900}{4} - 225 = 375$$

$$3) 300Y - 225 = 375$$

$$2) 100Y - 75 = 125$$

$$4Y - 3 = 5$$

$$4Y^2 - 3Y = 5Y$$

$$4Y^2 - 5Y = 3$$

$$Y^2 - 5Y - 3 = 0 \Rightarrow Y = \frac{5 \pm \sqrt{25 + 12}}{2} = \frac{5 \pm \sqrt{37}}{2}$$

$$Y = \frac{5 + \sqrt{37 + 12}}{2} = \frac{5 + \sqrt{49}}{2} = \frac{5 + 7}{2} = 6$$

$$\sqrt{256} = 16$$

1693/3000000 177.2 - B's land - 3300000 - 1.693 dollars per acre
 1693 122.8 A's land 3000000 - 2.443 - in an
 13079 750 diff per acre

177.2 B's 177.2 land
 3386 in 1693 price per acre
 44 5320 - 4 acres

15948 122.8
 10432 2.443
 1772 3684
 53000000 4912

2456
 53000000

A's Land 122.8 2.443 = 3000000

B's - 177.2 - 1.693 = 2999996
 750 diff

17.08800749 $\frac{5}{8}$
 16
 13670405992
 21670405992 B's price more exact
 125 21670405992 (1693000468 = X
 125
 588
 1132
 1132
 384
 384
 0000599
 512
 872
 748
 1940
 1940
 16

292 17.08800749 0.63474
 27192 26156
 1189 6156

34030000
 3403 27264
 34168 273600
 34168 273344
 341760256000

136 128 17.088 - 5
 88 16 X 8
 128 216 (1.69
 125
 880 128 216704 1.692 = Y A's val
 768 128 128 in 1693
 1120 128 128 in 1693
 1120 128 128 in 1693
 68 1120 1120
 384
 384

34176007256000000
 239232049
 341760146126785100
 41367020572
 341760158913275452400
 30758414301
 21763809900

Ratio 307 to 443

Problems - by Thomas Barlow 2^d - June 13th 1829

1.

If the square root of any number be multiplied by 2, 3, 4 &c. the product will be equal to the square ^{the product of} root of said Number multiplied by the square of said multiplier - -

Ex. Let 100 be the number & 16 its square root multiplied by $\frac{4}{40}$ which is equal to the square root of the product of $100 \times 16 = \sqrt{1600} = 40$ -

Prob. 2^d By Division - same - reversed -

Prob. 3^d

If any number whose square root is involved with itself is required - Rule - Take the square root by sub^d last quot. fig^s &c

Ex. Let the square ^{root} of a number added to itself be 56 - what is that num^r? $\frac{56}{2} \div 2 = 7 \times 7 = 49$ Answer
(added to)

If any number (with) the square root of 2, 3, 4 &c times its ^{product} number, be divided by 2, 3, 4 &c the quotient will be the number with its root - which root (extracted) multiplied by 2, 3, 4 &c will give the root ~~required~~ - or number required -

~~Ex.~~ A man being asked how many Dollars he had in his Pocket and if the number be multiplied by 4 - & the square root of the product be added to the number it will make 440 -

1600
80
1728
688
64
163488
480

6/1600
400 20
400
400
800 x 80 = 6400

$\sqrt{4x} = 440 - x$
 $4x = 193600 - 880x + x^2$
 $x^2 - 884x = -193600$

Let 440 be divided by 4
110 | 10 greater sq^r & 400 round.
20 | 10
40 root added to 400 = 440
& 400 the number

$\sqrt{4x + x} = 440$
 $4x + x = 440^2$
 $4x = 884x - 781456$
 $x = 884 - \frac{781456}{4}$
 $x = 800 \div 2 = 400$
The Number

$4x^2 - 3536x + 781456 = -774400 + 781456$
 $2x = 884 - \frac{7056}{84}$
 $x = 800 \div 2 = 400$
The Number

7056
84
164 656
4 656

Prob. 5th

When any Number with 2. 3. 4. &c. times its square root is given to find its root or Number — Rule

Divide the given Number by the square of the 2. 3. 4. &c. by which it has been increased & the quotient will be the number with its root involved, which root, multiplied by said square will give the root required

Ex. 1.

Prob. 6th

To form a Square within any given Number, that shall be equal to $\frac{1}{2}$. $\frac{1}{3}$. $\frac{1}{4}$. &c. of the Remainder — Rule

Multiply the whole Number by 2. 3. 4. &c. then extract the square root, by adding the last quotient ^{fig.} (by Prob. 3rd) which root divided by said Multiplier gives the square required.

The product of the sum & difference of any two numbers, is equal to the difference of their Squares. And if such numbers have a difference of but ONE

~~The sum of any two numbers multiplied by their difference the difference of their squares will be equal to their sum.~~
~~if two. it is equal to the difference of their squares.~~ { Aug. 6. 1537
if two. it is twice their sum; if 3. to 3 times &c. &c.

Twice the sum of the squares of any two numbers, exceeds the square of their sum — by the square of their difference.

The sum of the squares of any two numbers exceeds double their product, by the square of their difference. Crute Mar. Anno 16

The square of the leg of any right angled triangle exceeds the square of the hypotenuse, by four times the area of the triangle.

A tree one Hundred feet high stands by the side of a stream fifty feet wide - How far from its top must it be cut off so that it will just reach the opposite bank?

Rule.

Divide $\frac{1}{2}$ the sum of the Squares (of the height of the tree & the breadth of the stream) by the Height of the tree - Or, The whole sum, by twice the Height - The Quotient will be the Answer. The difference between the $\frac{1}{2}$ sum & the square of the height of the tree, divided by the height, will give the length of the stump.

Example. Algebra

Let x = Dist from top of the tree to the top of the stump

Then x being the Hyp. of a right ang. Δ

x^2 = the square of the 2 sides $60^2 + 100^2$

$60^2 + 100^2 = x^2$ the last exp. and gives

$10000 - 200x + x^2 = 3600$

$x^2 = 13600 - 200x + x^2$ Expanding on the side

\therefore Transposing $-200x$

$200x = 13600$

$x = 13600 \div 200 = 68$ the Hgt.

| | | |
|--------|--------|--------|
| 32 | 60 | 68 |
| 32 | 60 | 68 |
| 64 | | 544 |
| 96 | 3600 | 4624 |
| 1024 | 1024 | 4624 |
| | 4624 | |

Theorem.

The Hypotenuse in any right angle triangle is equal to the square of its base, together with the square of the sum of its other leg & itself, divided by twice said sum. Or, It is equal to Half the sum of the squares, of the base, & of its other leg together with itself, divided by the sum of said other leg & itself.

2d

Half the sum of any two squares, divided by a side of the greater, is equal to the Hypotenuse of a right angle triangle, whose base is a side of the less.

Index.

| | |
|--------------------------------------|----|
| Aut ^h Lang. Syne. | 4 |
| Ambassadors minuet. | 6 |
| Build Robin Gray. | 8 |
| Adams Hornpipe. | 8 |
| Augustines Waltz. | 18 |
| American Rush on. | 24 |
| Albany Volunteers March. | 26 |
| American Eagle. | 28 |

| | |
|-----------------------------------|----|
| Durandarte & Belerma. | 8 |
| Duke of Yorks quick step. | 18 |
| Devils Dream. | 20 |
| Double Drag. | 22 |
| Duke of Yorks March. | 28 |
| Durges Hornpipe. | 28 |
| Dunklin House. | 30 |
| Dutchells & Yorks Waltz. | 49 |

| | |
|---------------------------------|----|
| Blewetts Jig. | 48 |
| Bonny Lads Fisherow. | 40 |
| Butchers Round. | 38 |
| Blue eyed Mary. | 4 |
| Blodgets Hymn. | 12 |
| Bonny Lads of Aberdeen. | 30 |
| Blazing Star. | 34 |
| Bank of Flowers. | 24 |

| | |
|--------------------------------|---------|
| Fishers Hornpipe. | 30. 34. |
| French National March. | 32 |
| Free Masons March. | 32. |

| | |
|------------------------------------|---------|
| Copenhagen Waltz. 44. | |
| Coming through the Rye. | 1 |
| Cottage on the Moor. | 10. |
| Contented Cottager. | 12 |
| Constitution March. | 26 |
| Come haste to the Wedding. | 26 |
| College Hornpipe. | 30 |
| Cowden Broom. | 34 |
| Charter Castle. | 38 |
| Canandaigua Assembly. | 38 |
| Cameronian Rant. | 44 & 42 |

| | |
|--|-----|
| Green Sleeves with Variations. | 16. |
| Grand Spy. | 22 |
| Green Joke. | 26 |
| Genl Gourrs March. | 28. |

| | |
|-----------------------------|----|
| Hungarian Waltz. | 46 |
| Humours of Panteen. | 14 |
| Hail Columbia. | 24 |
| Haymakers. | 24 |
| Highland Mary. | 1 |
| How blithe be. | 2 |

Jesse the Flower & Dunblane - - - 4
 Jackson's Welcome Home - - - 8
 Jack on the Green - - - 10
 Irish Lilt - - - 14
 Irish Washer Woman - - - 46

Lads in yon Town - - - 49
 Long Hills of Morant - - - 40
 Logan Water - - - 30

Maid of Lodi - 36
 Meeting of the Waters - - - 2
 Mollonny's Jig - - - 6
 Mary's Dream - - - 12
 Miss Murray's Waltz - - - 16
 More's Rant - - - 32
 Mr. Cloud's Reel - - - 46

Northampton March - - - 30
 New rigged Ship - - - 14
 No. neither &c - - - 12
 New Century Hornpipe - - - 32

Oft in the stillly night. - - - 8

Peep of Day - - - 38
 Priest in his boots - 10
 Port Gordon - - - 2
 Prince Regent - - - 12
 Prince Leopold's Waltz - 18
 Paddy Carey - - - 52

Robin - - - 40
 Roy's Wife - - - 2
 Robin Adair - - - 14
 Revelly - - - 22
 Retreat - - - 22
 Roslin Castle - 26

Sweet Annie - - - 1
 Sweet is the Vale. - - - 6
 Sandy & Jenny. - - - 8
 Sally &c - - - 10
 Scotch Air - - - 14
 Speed the Plough - - - 23
 Soldiers Delight - - - 30
 Swiss Waltz - - - 32
 Soldier's Joy - - - 24
 Soldier's Bride - - - 38
 Sheela na Guira - 46

Trip to Holland 37.
 nth Regt March 8
 Take care you lose 10

Vex'd Editor - - - 18

Tyrolese Waltz - 53

Waltz. n^o 1. - - - 4
 Do. - - - n^o 2. - - -
 Wars Alarms - - - 10
 Washington 6th M. 22.
 White Cockade - - - 24
 Well done Jack - - - 36
 Whistle &c - - - 46

York Fusilier - - - 32
 Yellow haired Laddie - - - 1
 Yankee Doodle - - - 20

Rickett's Reel - - - 40
 Do. Hornpipe - - - 40

$$x + \sqrt{4x} = 441 \quad \text{Completing the square}$$

$$x + \sqrt{4x} + \frac{1}{4} = 441 + \frac{1}{4}$$

$$\text{Radicals } x = -\frac{1}{2} \pm \sqrt{440 + \frac{1}{4}}$$

$$x = 17.61$$

$$81 \overline{) 17.61}$$

$$17.61 \overline{) 41.964}$$

$$20.482$$

$$4 \times 4 = 16$$

$$2 \times 4$$

$$x + 2\sqrt{x} + 1 = 441 \quad \text{Completing square}$$

$$\sqrt{x} = -1 \pm \sqrt{441} \quad (21 - 1 = 20)$$

$$41 \overline{) 41}$$

$$41 \overline{) 41}$$

$$\sqrt{x} = 20$$

$$x = 400$$

$$8386.83900$$

$$8386.83900$$

$$83924.338608$$

$$4 \overline{) 332896}$$

$$82704$$

David Beatty for himself & heirs
\$11.43

